Warm Up

1. Describe the similarities and differences between the graphs of $f(x)$ and $g(x)$.

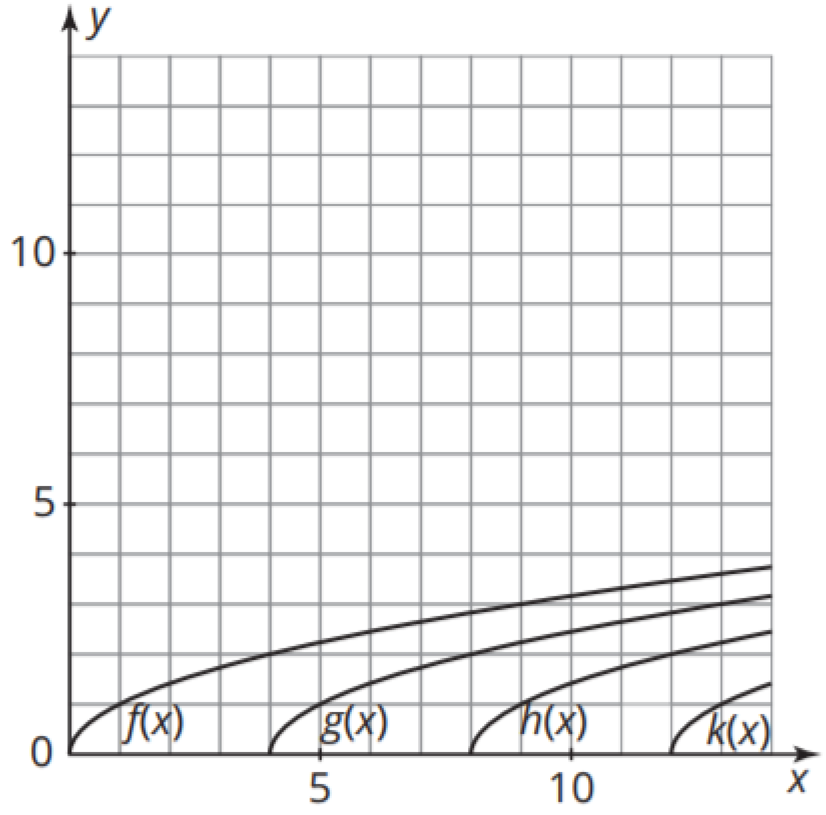
   $f(x) = h(x - 5)$

   $g(x) = h(x) + 5$

A Sea Change

A group of art students had the idea to use transformations of radical functions to create a logo for the Radical Surfing School.

To start, they graphed the function $f(x) = \sqrt{x}$, for $0 \leq x \leq 14$, and shifted copies of the curve to create the waves $g(x)$, $h(x)$, and $k(x)$.



Remember:

The transformation function form is $g(x) = Af(B(x - C)) + D$.

1. **What value(s) in the transformation function form were changed to create these curves? Explain your reasoning.**

$$f(x) = A\sqrt{B(x-C)} + D$$

2. **Do the graphs of *g*(*x*), *h*(*x*), and *k*(*x*) have the same domain as *f*(*x*)? Explain your reasoning.**

Ask yourself:

Do the transformations of *f*(*x*) shown on the graph take place inside the function or outside the function?

Consider the transformations of the radical function $f(x) = \sqrt{(x)}$ graphed by the art students in the previous activity.

1. Devin, Stuart, and Kristen each wrote an equation for one of the functions that was added to the graph, first in terms of $f(x)$ and then in terms of x.

- **Devin's equation:** $g(x) = f(x) - 4$
  $= \sqrt{x} - 4$
- **Stuart's equation:** $h(x) = f(x - 8)$
  $= \sqrt{x - 8}$
- **Kristen's equation:** $k(x) = f(x + 12)$
  $= \sqrt{x + 12}$

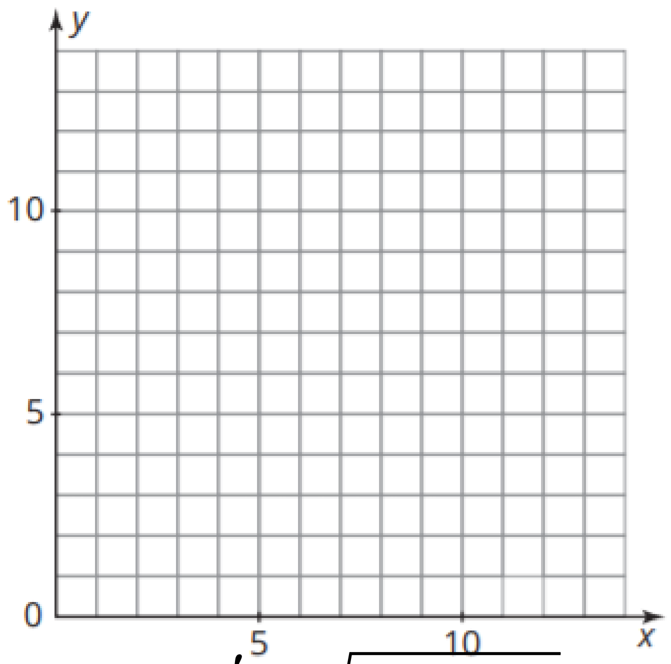
Explain why each student's equation is either correct or incorrect. If it is incorrect, write the correct equation, first in terms of $f(x)$ and then in terms of x. Finally, state the domain of each.

The students decide that reflecting each curve, $g(x)$, $h(x)$, and $k(x)$, across the respective lines where $x = C$ will make them look more like waves crashing on the beach.

$$f(x) = A\sqrt{B(x - C)} + D$$

2. Consider the graph of the reflections.

a. Graph the resulting functions $f'(x)$, $g'(x)$, $h'(x)$, and $k'(x)$. Write each function, first in terms of its transformation of $f(x)$, $g(x)$, $h(x)$, or $k(x)$, and then in terms of x. Finally, state the domain of each.



Remember:

You can use the prime symbol (′) to indicate that a function is a transformation of another function.

$$f(x) = \sqrt{-(x - C)}$$

$$f(x)' = \sqrt{-x}$$

$$g(x)' = \sqrt{-(x - 4)}, [0,4]$$

$$h(x)' = \sqrt{-(x - 8)}, [0,8]$$

$$k(x)' = \sqrt{-(x - 12)}, [0,12]$$

b. Describe how you can use the transformation function form to determine the equations of the new functions.

A Reflection over C, means the B value is negative 1

c. How did the domain of each transformed function change as a result of the reflection across $x = C$?

The Domain becomes the opposite of what it was before

d. Why does your graph show only 3 curves when the original graph had 4? Explain your reasoning.

$f(x) = \sqrt{-x}$ does not exist for positive x-values

Suppose the students wanted to reflect the 3 new waves $g'(x)$, $h'(x)$, and $k'(x)$ across the line $y = 0$.

$$f(x) = A\sqrt{B(x-C)} + D$$

3. Consider the equations for the reflected functions.

Use the double prime symbol (") to indicate each transformed function.

a. Describe how you can use transformation function form to determine the equations of the reflected functions.

A Reflection over y=0, means the A value is negative 1

b. Write three new functions using transformational form to represent each reflection of $g'(x)$, $h'(x)$, and $k'(x)$, and then each in terms of x. Finally, write the domain of each transformed function.

$$g(x)'' = -\sqrt{-(x-4)}$$

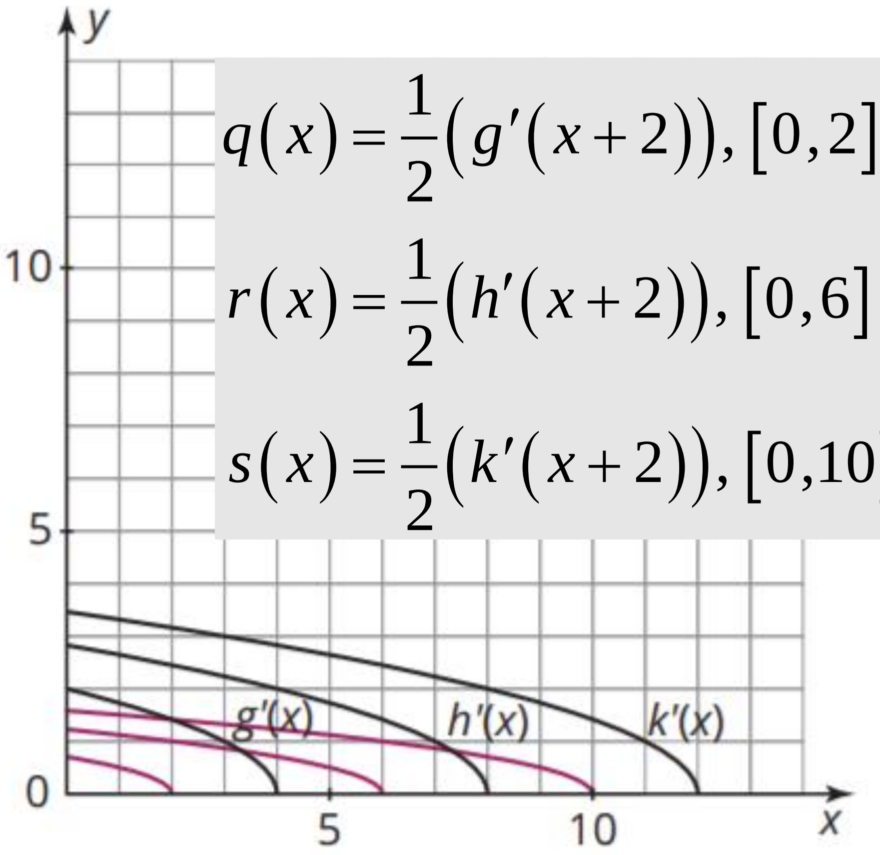
$$h(x)'' = -\sqrt{-(x-8)}$$

$$k(x)'' = -\sqrt{-(x-12)}$$

The art students want to add waves below the 3 waves, as shown. These waves are copies of $g'(x)$, $h'(x)$, and $k'(x)$, except half as high and shifted to the left 2 units.

4. Consider the equations for the waves.

a. Write 3 new functions $q(x)$, $r(x)$, and $s(x)$ in terms of $g'(x)$, $h'(x)$, and $k'(x)$ to create the waves that the art students want. Make sure to write the domains of each transformed function.



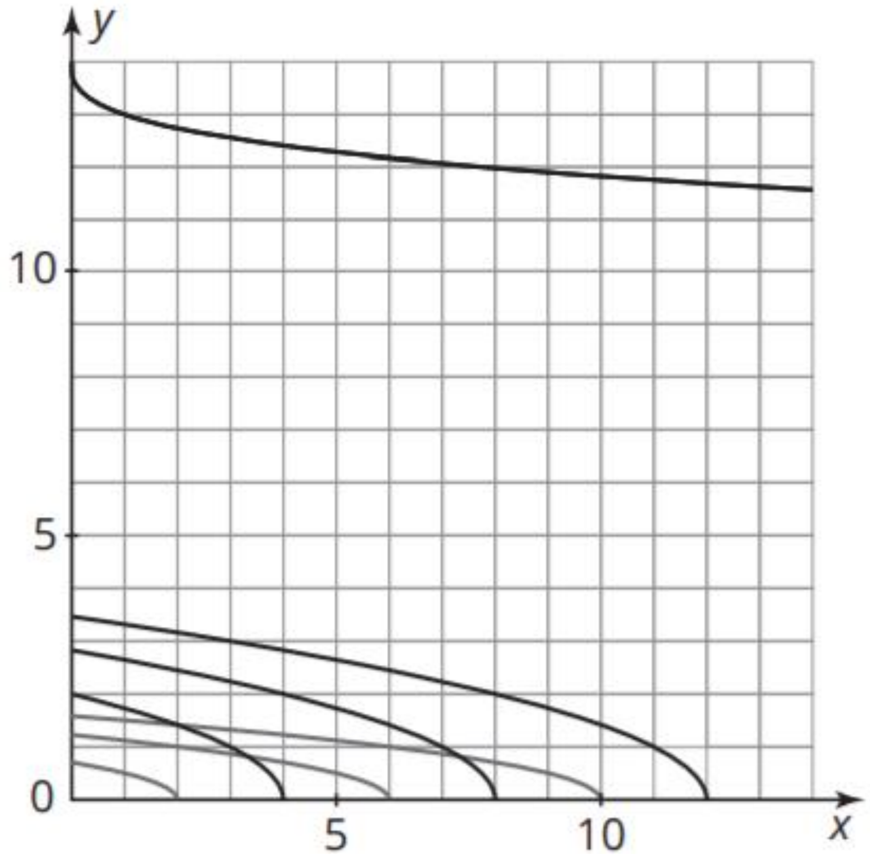
$$q(x)=\frac{1}{2}\left(g'(x+2)\right),\ [0,2]$$

$$r(x)=\frac{1}{2}\left(h'(x+2)\right),\ [0,6]$$

$$s(x)=\frac{1}{2}\left(k'(x+2)\right),\ [0,10]$$

The art students want to add some clouds to the top of the logo. For the clouds, they use the inverses of cubic functions. They start with the function $c(x) = -\sqrt[3]{x} + 14$.

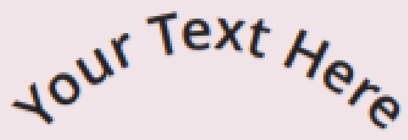
5. Transform this function and write 2 more equations to create the clouds the students want. Graph the results.



$$d(x) = c(x) - 1$$
$$= -\sqrt[3]{x} + 13$$

$$t(x) = c(x) - 2$$
$$= -\sqrt[3]{x} + 12$$

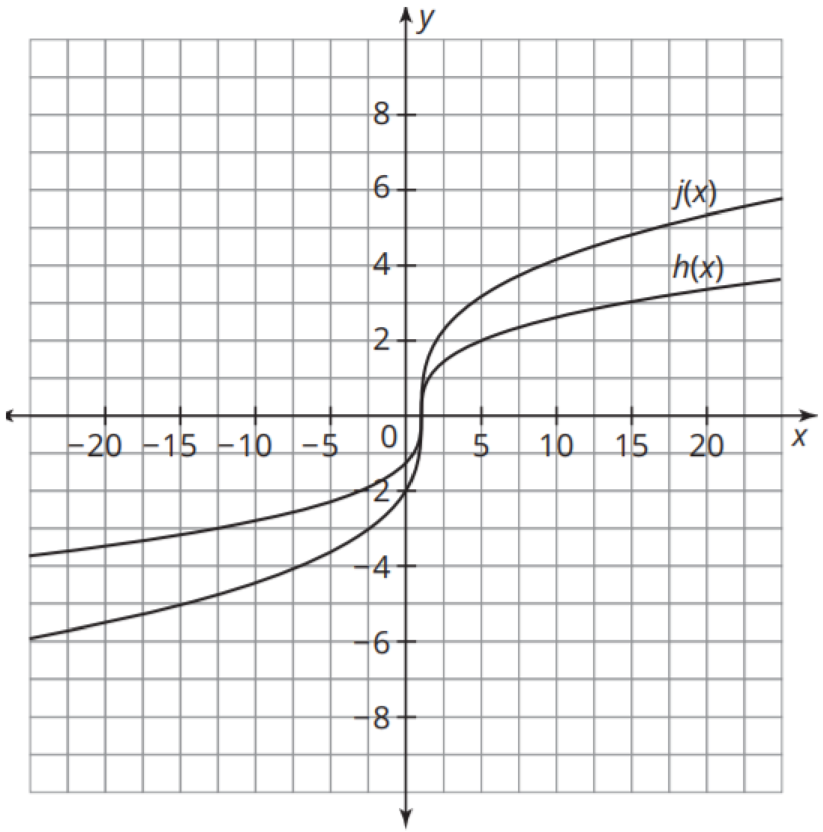
The text shown, for example, follows the curve $f(x) = -x^2$.



In many graphic design programs, a trace path can be created. A trace path is an invisible line or curve that acts as the baseline of text that is added to the design. When you insert text on a trace path, the text follows the line or curve.

6. **The art students are experimenting with different radical function graphs to use as trace paths for the surfing school's name: Radical Surfing School. They have narrowed their trace paths down to 2 choices. The graphs of the functions are shown.**

$h(x) = \sqrt[3]{2(x - 1)}$ $\qquad$ $j(x) = 2\sqrt[3]{x - 1}$



a. **Use technology to sketch the graph of the function $\sqrt[3]{x}$ and list its domain, range, and *x*-intercept and *y*-intercept.**

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

***x*-intercept:** $(0,0)$

***y*-intercept:** $(0,0)$

b. Compare and contrast the graphs of the functions $h(x)$ and $j(x)$ and their equations. What do you notice?

c. Compare the effects of increasing the *A*-value with increasing the *B*-value in a radical function. What do you notice?

d. State the domain of each transformed function.