Worked Example

You can rewrite an expression in radical form as an expression with a rational exponent. Solve the equation $\sqrt{x} = x^a$ for a, given $x \geq 0$, to determine the exponential form of $\sqrt{x}$.

$$\sqrt{x} = x^a$$

Square each side of the equation.

$$(\sqrt{x})^2 = (x^a)^2$$

The bases are the same, so set the exponents equal to each other, and solve for a.

$$x = x^{2a}$$

$$1 = 2a$$

$$a = \frac{1}{2}$$

The exponential form of the square root of x, given $x \geq 0$, is x to the one-half power.

$$\sqrt{x} = x^{\frac{1}{2}}, \text{ given } x \geq 0$$

1. **Why is the restriction "given $x \geq 0$" stated at the beginning of the worked example?**

2. **How do you know when the initial x-value can be any real number or when the initial x-value should be restricted to a subset of the real numbers?**

3. Rewrite each expression in radical form as an expression with a rational exponent.

a. The cube root of *x*

b. The cube root of *x* squared

4. Complete the cells in each row. In the last column, write "$x \geq 0$" or "all real numbers" to describe the restrictions that result in equal terms for each row.

Radical Form	Radical to a Power Form	Exponential Form	Restrictions
$\sqrt[4]{x^2}$	$(\sqrt[4]{x})^2$		
		$x^{\frac{3}{4}}$	
		$x^{\frac{2}{5}}$	
$\sqrt[5]{x}$			

You can rewrite a radical expression $\sqrt[n]{x^a}$ as an exponential expression $x^{\frac{a}{n}}$:

- For all real values of x if the index n is odd.
- For all real values of x greater than or equal to 0 if the index n is even.

Worked Example

Extract the roots and rewrite $\sqrt[3]{8x^6}$ using radicals and using powers.

Using Radicals

$$\begin{aligned}\sqrt[3]{8x^6} &= \sqrt[3]{2^3 \cdot x^6}\\ &= \sqrt[3]{2^3 \cdot (x^2)^3}\\ &= \sqrt[3]{2^3} \cdot \sqrt[3]{(x^2)^3}\\ &= 2x^2\end{aligned}$$

Using Powers

$$\begin{aligned}\sqrt[3]{8x^6} &= (8x^6)^{\frac{1}{3}}\\ &= (2^3 \cdot x^6)^{\frac{1}{3}}\\ &= 2^{\frac{3}{3}} \cdot x^{\frac{6}{3}}\\ &= 2^1 \cdot x^2\\ &= 2x^2\end{aligned}$$

The root of a product is equal to the product of the roots of each factor: $\sqrt[p]{a^m b^n} = \sqrt[p]{a^m} \cdot \sqrt[p]{b^n}$.

1. Which method do you prefer?

2. Embry and Devon shared their work for extracting roots from $\sqrt[4]{j^8k^4}$.

Embry

$$\sqrt[4]{j^8k^4} = \sqrt[4]{j^8} \cdot \sqrt[4]{k^4}$$
$$= \sqrt[4]{(j^2)^4} \cdot \sqrt[4]{k^4}$$
$$= j^2|k|$$

Devon

$$\sqrt[4]{j^8k^4} = (j^8k^4)^{\frac{1}{4}}$$
$$= (j^8)^{\frac{1}{4}}(k^4)^{\frac{1}{4}}$$
$$= j^{\frac{8}{4}}k^{\frac{4}{4}}$$
$$= j^2|k|$$

Explain why it is not necessary to use the absolute value symbol around j^2 and why it is necessary to use the absolute value symbol around k.

3. Betty, Wilma, and Rose each extracted roots and rewrote the radical $\sqrt{x^2y^2}$.

Betty

$$\sqrt{x^2y^2} = \sqrt{x^2 \cdot y^2}$$
$$= \sqrt{x^2} \cdot \sqrt{y^2}$$
$$= |x| \cdot |y|$$

Wilma

$$\sqrt{x^2y^2} = \sqrt{x^2 \cdot y^2}$$
$$= \sqrt{x^2} \cdot \sqrt{y^2}$$
$$= |xy|$$

Rose

$$\sqrt{x^2y^2} = \sqrt{x^2 \cdot y^2}$$
$$= \sqrt{x^2} \cdot \sqrt{y^2}$$
$$= xy$$

Who's correct? Explain you reasoning.

4. Rewrite each radical involving a quotient.

a. $\sqrt{\dfrac{m^4}{n^6}}$

b. $\sqrt[3]{\dfrac{j^{12}}{k^3}}$

For some radicals, you may not be able to extract the entire radicand.

5. Bernadette and Cris extracted the roots from $\sqrt[3]{16x^8}$.

Bernadette

$$\sqrt[3]{16x^8} = 2^{\frac{4}{3}} \cdot x^{\frac{8}{3}}$$
$$= 2^{\frac{3}{3}} \cdot 2^{\frac{1}{3}} \cdot x^{\frac{6}{3}} \cdot x^{\frac{2}{3}}$$
$$= 2 \cdot \sqrt[3]{2} \cdot x^2 \cdot \sqrt[3]{x^2}$$
$$= 2x^2\sqrt[3]{2x^2}$$

Cris

$$\sqrt[3]{16x^8} = \sqrt[3]{2^4 \cdot x^8}$$
$$= \sqrt[3]{2^3x^6} \cdot \sqrt[3]{2x^2}$$
$$= 2x^2\sqrt[3]{2x^2}$$

a. In the last line of work, why was $2x^2$ not extracted from the radical?