#### Worked Example

You can rewrite an expression in radical form as an expression with a rational exponent. Solve the equation  $\sqrt{x} = x^{\alpha}$  for  $\alpha$ , given  $x \ge 0$ , to determine the exponential form of  $\sqrt{x}$ .

Square each side of the equation. 
$$\sqrt{x} = x^a$$

$$(\sqrt{x})^2 = (x^a)^2$$
The bases are the same, so 
$$x = x^{2a}$$
set the exponents equal 
$$1 = 2a$$
to each other, and solve for  $a$ . 
$$a = \frac{1}{2}$$

The exponential form of the square root of x, given  $x \ge 0$ , is x to the one-half power.

$$\sqrt{x} = x^{\frac{1}{2}}$$
, given  $x \ge 0$ 

1. Why is the restriction "given  $x \ge 0$ " stated at the beginning of the worked example?

2. How do you know when the initial x-value can be any real number or when the initial x-value should be restricted to a subset of the real numbers? 3. Rewrite each expression in radical form as an expression with a rational exponent.

a. The cube root of x

b. The cube root of *x* squared

4. Complete the cells in each row. In the last column, write " $x \ge 0$ " or "all real numbers" to describe the restrictions that result in equal terms for each row.

Radical Form	Radical to a Power Form	Exponential Form	Restrictions
$\sqrt[4]{X^2}$	$(\sqrt[4]{X})^2$		
		$\chi^{\frac{3}{4}}$	
		$\chi^{\frac{2}{5}}$	
5√ <i>X</i>			

You can rewrite a radical expression  $\sqrt[n]{x^a}$  as an exponential expression  $x^{\frac{n}{n}}$ :

- For all real values of x if the index n is odd.
- For all real values of x greater than or equal to 0 if the index n is even.

## Worked Example

Extract the roots and rewrite  $\sqrt[3]{8x^6}$  using radicals and using powers.

## **Using Radicals**

$$\sqrt[3]{8x^6} = \sqrt[3]{2^3 \cdot x^6} 
= \sqrt[3]{2^3 \cdot (x^2)^3} 
= \sqrt[3]{2^3 \cdot \sqrt[3]{(x^2)^3}} 
= 2x^2$$

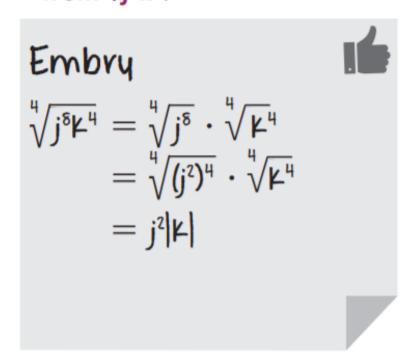
## **Using Powers**

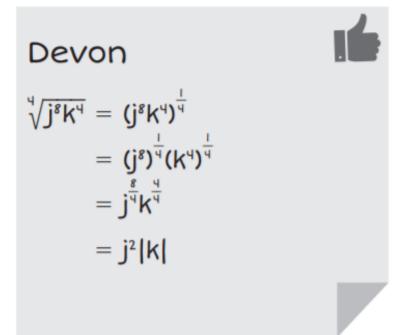
$$\sqrt[3]{8x^6} = (8x^6)^{\frac{1}{3}} 
= (2^3 \cdot x^6)^{\frac{1}{3}} 
= 2^{\frac{3}{3}} \cdot x^{\frac{6}{3}} 
= 2^1 \cdot x^2 
= 2x^2$$

The root of a product is equal to the product of the roots of each factor:  $\sqrt[p]{a^m b^n} = \sqrt[p]{a^m \cdot \sqrt[p]{b^n}}$ .

## 1. Which method do you prefer?

# 2. Embry and Devon shared their work for extracting roots from $\sqrt[4]{j^8k^4}$ .





Explain why it is not necessary to use the absolute value symbol around  $j^2$  and why it is necessary to use the absolute value symbol around k.



3. Betty, Wilma, and Rose each extracted roots and rewrote the radical  $\sqrt{x^2y^2}$ .

Betty
$$\sqrt{x^2y^2} = \sqrt{x^2 \cdot y^2}$$

$$= \sqrt{x^2} \cdot \sqrt{y^2}$$

$$= |x| \cdot |y|$$
Wilma
$$\sqrt{x^2y^2} = \sqrt{x^2 \cdot y^2}$$

$$= \sqrt{x^2} \cdot \sqrt{y^2}$$

$$= |xy|$$
Rose
$$\sqrt{x^2y^2} = \sqrt{x^2 \cdot y^2}$$

$$= \sqrt{x^2} \cdot \sqrt{y^2}$$

$$= |xy|$$

$$= |xy|$$

Wilma

$$\sqrt{x^2y^2} = \sqrt{x^2 \cdot y^2} 
= \sqrt{x^2} \cdot \sqrt{y^2} 
= |xy|$$

Rose

$$\sqrt{x^2 y^2} = \sqrt{x^2 \cdot y^2} 
= \sqrt{x^2} \cdot \sqrt{y^2} 
= xy$$

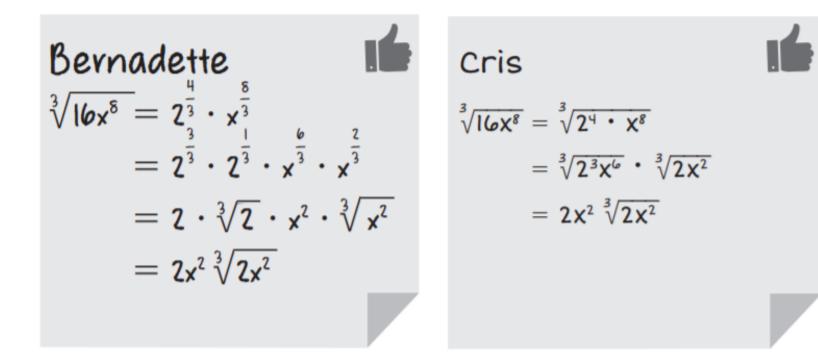
Who's correct? Explain you reasoning.

# 4. Rewrite each radical involving a quotient.

a. 
$$\sqrt{\frac{m^4}{n^6}}$$

b. 
$$\sqrt[3]{\frac{j^{12}}{k^3}}$$

#### 5. Bernadette and Cris extracted the roots from $\sqrt[3]{16x^8}$ .



a. In the last line of work, why was  $2x^2$  not extracted from the radical?