6. Rewrite each radical by extracting all possible roots, and write the final answer in radical form.

a. $\sqrt{16x^6}$

b. $-\sqrt{8v^3}$

c. $\sqrt{d^3f^4}$

d. $\sqrt{h^4j^6}$

When the power and root are equal and even numbers, remember to use absolute value for the principal root.

e. $\sqrt{25a^2b^8c^{10}}$

f. $\sqrt[4]{81x^5y^{12}}$

g. $\sqrt[3]{(x+3)^9}$

h. $\sqrt{(x+3)^2}$

When the power and root are equal and even numbers, remember to use absolute value for the principal root.

Arianna and Heidi multiplied $\sqrt{18a^2} \cdot 4\sqrt{3a^2}$ and extracted all roots.

1. Compare Arianna's and Heidi's solution methods. Explain the difference in their solution methods.

Arianna

$$\begin{aligned}\sqrt{18a^2} \cdot 4\sqrt{3a^2} &= 4\sqrt{54a^4} \\ &= 4\sqrt{9 \cdot 6 \cdot a^4} \\ &= 4 \cdot \sqrt{9} \cdot \sqrt{6} \cdot \sqrt{a^4} \\ &= 4 \cdot 3 \cdot \sqrt{6} \cdot a^2 \\ &= 12a^2\sqrt{6}\end{aligned}$$

Heidi

$$\begin{aligned}\sqrt{18a^2} \cdot 4\sqrt{3a^2} &= \sqrt{9 \cdot 2 \cdot a^2} \cdot 4 \cdot \sqrt{3 \cdot a^2} \\ &= \sqrt{9} \cdot \sqrt{2} \cdot \sqrt{a^2} \cdot 4 \cdot \sqrt{3} \cdot \sqrt{a^2} \\ &= 3 \cdot \sqrt{2} \cdot |a| \cdot 4 \cdot \sqrt{3} \cdot |a| \\ &= 12a^2\sqrt{6}\end{aligned}$$

You have seen that the root of a product is equal to the product of the roots of each factor. This concept applies to quotients also. The root of a quotient is equal to the quotient of the roots of the dividend and divisor: $\sqrt[p]{\frac{a^m}{b^n}} = \frac{\sqrt[p]{a^m}}{\sqrt[p]{b^n}}$.

Worked Example

Extract the roots to rewrite the quotient $\frac{\sqrt{18a^2}}{4\sqrt{3a^2}}$.

$$\frac{\sqrt{18a^2}}{4\sqrt{3a^2}} = \frac{\sqrt{6 \cdot \cancel{3} \cdot \cancel{a^2}}}{4\sqrt{\cancel{3}\cancel{a^2}}}$$

$$= \frac{\sqrt{6}}{4}$$

3. Jackie shared her solution for extracting roots and rewriting the quotient $\frac{\sqrt{25bc}}{\sqrt[3]{b^2c^2}}$, given $b > 0$ and $c > 0$. Explain why Jackie's work is incorrect.

Jackie

$$\frac{\sqrt{25bc}}{\sqrt[3]{b^2c^2}} = \frac{\sqrt{25bc}}{\sqrt[3]{b^2c^2}}$$

$$= \frac{\sqrt{25}}{\sqrt[3]{bc}}$$

$$= \frac{5}{\sqrt[3]{bc}}$$

4. Perform each operation and extract all roots. Write your final answer in radical form.

a. $2\sqrt{x} \cdot \sqrt{x} \cdot 5\sqrt{x}$, given $x \geq 0$

b. $2(\sqrt[3]{k})(\sqrt[3]{k})$

Ask yourself:

Why are the restrictions $b > 0$ and $c > 0$, instead of $b \geq 0$ and $c \geq 0$?

c. $7\sqrt{h}(3\sqrt{h} + 4\sqrt{h^3})$, given $h \geq 0$

d. $\sqrt{a} \cdot \sqrt[3]{a}$, given $a \geq 0$

e. $(n)(\sqrt[3]{4n})(\sqrt[3]{2n^2})$

f. $\sqrt{\frac{4x^4}{x^2}}$, given $x \neq 0$

5. Consider how Grace, Diane, Ron, and Sheila rewrote the sum or difference of their original expression as one term.

Grace

$2\sqrt{x} + 6\sqrt{x} = 8\sqrt{x}$

Diane

$16\sqrt[3]{x^2} - 10\sqrt[3]{x^2} = 6\sqrt[3]{x^2}$

Ron

$0.1\sqrt{x} + 3.6\sqrt[3]{x} = 3.7\sqrt[3]{x}$

Sheila

$\sqrt{x} + \sqrt{y} = 2\sqrt{xy}$

Worked Example

To determine the sum or difference of like radicals, add or subtract the coefficients.

$$3\sqrt{x} + \sqrt{x} = 4\sqrt{x}\text{, given } x \geq 0$$

You can also write an equivalent expression using powers.

$$3x^{\frac{1}{2}} + x^{\frac{1}{2}} = 4x^{\frac{1}{2}}\text{, given } x \geq 0$$

6. **Larry considered whether or not $4\sqrt{x}$ and $-5x^{\frac{1}{2}}$ are like terms, given $x \geq 0$.**

Larry

They are not like terms because they are not written in the same form.

Explain the error in Larry's reasoning.

7. Combine like terms, if possible, and write your final answer in radical form.

a. $\sqrt{y} - \sqrt{y}$, **given** $y \geq 0$

b. $9\sqrt{a} + 5\sqrt{b}$, **given** $a \geq 0$, $b \geq 0$

c. $2\sqrt{x} + \sqrt{x} + 5\sqrt{x}$, **given** $x \geq 0$

d. $7\sqrt{h} - 4.1\sqrt{h} + 2.4\sqrt{h}$, given $h \geq 0$

e. $3\sqrt{t}\,(\sqrt{t} - 8\sqrt{t}) + 4t$, given $t \geq 0$

f. $5\sqrt{g} + 2\sqrt[3]{g}$, given $g \geq 0$