Warm Up

Determine the value of *x*, if possible.

1. $\sqrt{x} = 2$

2. $\sqrt[3]{x} = -2$

3. $\sqrt{x} = -2$

Learning Goals

- Use Properties of Equality to solve radical equations.
- Identify extraneous roots when solving radical equations.

Key Term

- extraneous solutions

Worked Example

Solution Steps for a Quadratic Equation

$$2x^2 - 5 = 13$$
$$2x^2 = 18$$
$$x^2 = 9$$
$$\sqrt{x^2} = \sqrt{9}$$
$$x = \pm 3$$

Solution Steps for a Radical Equation

$$2\sqrt{x} - 5 = 13$$
$$2\sqrt{x} = 18$$
$$\sqrt{x} = 9$$
$$(\sqrt{x})^2 = (9)^2$$
$$x = 81$$

1. Analyze the examples.

a. Describe the similarities in the first two steps of each solution.

b. Describe the differences in the remaining steps of each solution.

2. How would the strategy shown in the worked example change for cube and cube root equations? Provide an example to explain your reasoning.

Worked Example

Solve $3\sqrt{x} + 7 = 25$.

$$3\sqrt{x} + 7 = 25$$
$$3\sqrt{x} = 18$$
$$\sqrt{x} = 6$$
$$(\sqrt{x})^2 = (6)^2$$
$$x = 36$$

Check:

$$3\sqrt{36} + 7 \stackrel{?}{=} 25$$
$$3(6) + 7 \stackrel{?}{=} 25$$
$$25 = 25 \checkmark$$

Solve $\sqrt{x + 1} = 8$.

$$\sqrt{x + 1} = 8$$
$$(\sqrt{x + 1})^2 = (8)^2$$
$$x + 1 = 64$$
$$x = 63$$

Check:

$$\sqrt{63 + 1} \stackrel{?}{=} 8$$
$$\sqrt{64} \stackrel{?}{=} 8$$
$$8 = 8 \checkmark$$

You should always check your answers when solving equations. But with radical equations, it's extra important to check your answers. You'll soon learn why.

1. **Analyze the worked examples.**

   a. **How does the form of the equation $3\sqrt{x} + 7 = 25$ compare to the form of the equation $\sqrt{x + 1} = 8$?**

   b. **How does the form of each equation relate to the solution strategy shown?**

Raising both sides of an equation to a power may introduce an *extraneous solution*, so it is important to check your answers.

Extraneous solutions are solutions that result from the process of solving an equation but are not valid solutions to the equation.

2. Solve and check each equation.

a. $\sqrt{2x} = 3$

b. $\sqrt[3]{2x - 3} = 2$

c. $4\sqrt{x - 6} = 8$

d. $\sqrt{2x + 1} = 5$

e. $2\sqrt[3]{x} + 16 = 0$

f. $\sqrt{3x - 1} + 9 = 8$

3. Marteiz solved the equation $x - \sqrt{x} = 2$. Explain Marteiz's error. Give the correct solution in your explanation.

Marteiz

$$x - \sqrt{x} = 2$$
$$-\sqrt{x} = -x + 2$$
$$\sqrt{x} = x - 2$$
$$(\sqrt{x})^2 = (x - 2)^2$$
$$x = x^2 - 4x + 4$$
$$0 = x^2 - 5x + 4$$
$$0 = (x - 4)(x - 1)$$
$$x = 4 \text{ or } x = 1$$

The solution is $x = 4$ or $x = 1$.

4. Solve the equation $x - 1 = \sqrt{x + 1}$.

The Beaufort scale is a system that measures wind speed and describes conditions at sea and on land. The scale's range is from 0 to 12. A zero on the Beaufort scale means that the wind speed is less than 1 mile per hour and the conditions at sea and on land are calm. A twelve on the Beaufort scale represents hurricane conditions with wind speeds greater than 74 miles per hour, resulting in greater than 50-foot waves at sea and severe damage to structures and landscape.

1. Consider the equation $V = 1.837B^{\frac{3}{2}}$ that models the relationship between wind speed in miles per hour V and the Beaufort numbers B.

a. Solve the equation for B.

b. Determine the Beaufort number for a wind speed of 20 miles per hour.

In medicine, Body Surface Area *BSA* is used to help determine proper dosage for medications.

2. The equation $BSA = \frac{\sqrt{W \cdot H}}{60}$ models the relationship between *BSA* in square meters, the patient's weight *W* in kilograms, and the patient's height *H* in centimeters.

a. Solve the equation for *H*.

b. Determine the height of a patient who weighs 90 kilograms and has a *BSA* of 2.1.