Warm Up

Determine each product or quotient. Write each answer with the given base and a single exponent.

1. $(2^3)(2^5)$

2. $\frac{2^4}{2^8}$

3. $\frac{5^5}{5^4}$

4. $4^0 \cdot 4^1$

Let's consider another scenario that can have integer or fractional intervals.

In a laboratory experiment, a certain bacteria doubles each hour.

1. Suppose a bacteria population starts with just 1 bacterium.

a. Complete the table to show the population of bacteria, *f*(*x*), over time, *x*.

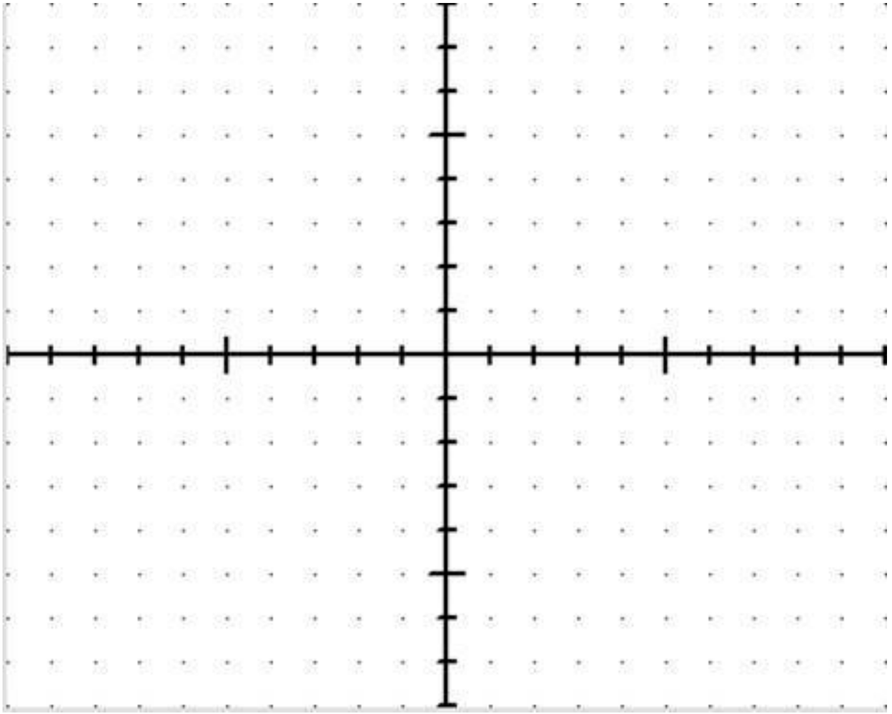
x	1	2	3	4
$f(x)$				

b. Determine the constant ratio and *y*-intercept. Then write the exponential function that represents the growth of the bacteria population over time. Show your work.

The population sequence is a geometric sequence, which has a constant ratio between terms. The constant ratio is a multiplier. To determine the next term of a geometric sequence, you multiply by this value.

c. **How is the constant multiplier evident in the problem situation?**

2. **Graph the exponential function to show bacteria growth over time on the coordinate plane located at the end of the lesson.**



The table shown represents the function $f(x) = 2^x$, which models the laboratory experiment that a certain population of bacteria can double each hour.

x	0	1	2	3
$f(x)$	2^0	2^1	2^2	2^3
	1	2	4	8

In the table, the interval between the input values is 1, and the constant multiplier is 2 at the point when the interval changes. What effect, if any, is there on the constant multiplier if the input interval is different?

1. Consider the ratio $\frac{f(2)}{f(0)}$.

a. Describe the interval of input values. Then determine the multiplier.

b. Write two additional ratios that have the same multiplier. Explain your reasoning.

Vicky, Nate, and Taylor are interested in the population of bacteria at each $\frac{1}{2}$-hour interval. They have values for the exponential function $f(x)$ when x is an integer. They need the values of the exponential function when x is a rational number between integers.

2. The three students used the idea of the constant multiplier to estimate the value of $f\left(\frac{1}{2}\right)$ for the function $f(x) = 2^x$.

Vicky

I know the constant multiplier for an interval of 1 is 2.
I want to split each interval of 1 into two equal parts, which means I need two equal multipliers.

multiply by 2

x	0	$\frac{1}{2}$	1
$f(x)$	2^0	$2^{\frac{1}{2}}$	2^1
	1		2

$1 \cdot r \cdot r = 2$

So, $r^2 = 2$.

Nate

If r is a constant multiplier for the function as it grows by consecutive integers, it can be split into two equal multipliers of $\sqrt{r}$, because r can be split into two equal factors of $\sqrt{r}$.

$$(\sqrt{r})^2 = r$$

Taylor

If $f(0) = 2^0 = 1$ and
$f(1) = 2^1 = 2$, then
$f\left(\frac{1}{2}\right) = 2^{\frac{1}{2}}$ must be equal to 1.5.

a. Use Vicky's and Nate's thinking to determine $f\left(\frac{1}{2}\right)$. Write $f\left(\frac{1}{2}\right)$ as a power of 2 and in radical form. Then, enter the values in the table.

x	0	$\frac{1}{2}$	1	2	3
$f(x)$	2^0		2^1	2^2	2^3
	1		2	4	8

b. Use the graph you created in the previous activity to approximate $f\left(\frac{1}{2}\right)$ as a decimal.

The expression $\sqrt{2} = 2^{\frac{1}{2}}$. The square root symbol ($\sqrt{\ }$) is interpreted as the rational exponent $\frac{1}{2}$. All the properties with integer exponents you previously learned continue to apply even when the exponent is a rational number.

The tables shown represent two different equivalent representations of the constant multiplier, $2^{\frac{1}{2}}$ or $\sqrt{2}$, for the function $f(x) = 2^x$.

	Rational Exponent Representation
$f(0)$	2^0
$f\left(\frac{1}{2}\right)$	$2^{\frac{1}{2}}$
$f(1)$	2^1
$f\left(\frac{3}{2}\right)$	$2^{\frac{3}{2}}$

$2^0 \cdot 2^{\frac{1}{2}}$

$2^{\frac{1}{2}} \cdot 2^{\frac{1}{2}}$

$2^1 \cdot 2^{\frac{1}{2}}$

	Radical Form Representation
$f(0)$	1
$f\left(\frac{1}{2}\right)$	$\sqrt{2}$
$f(1)$	2
$f\left(\frac{3}{2}\right)$	$2\sqrt{2}$

$1 \cdot \sqrt{2}$

$\sqrt{2} \cdot \sqrt{2}$

$2 \cdot \sqrt{2}$

The number 2 is a rational number because it can be represented as the ratio of two integers. The number $\sqrt{2}$ is an irrational number, because it cannot be represented as the ratio of two integers.

3. Use the properties of exponents to justify that $2^1 \cdot 2^{\frac{1}{2}} = 2^{\frac{3}{2}}$. Then use the graph to estimate $2^{\frac{3}{2}}$ as a decimal.

A rational exponent can be rewritten in radical form using the definition $a^{\frac{1}{n}} = \sqrt[n]{a}$. When the index is 2, it is usually implied rather than written.

Let's consider the properties of exponents to rewrite expressions in equivalent forms.

In the expression $\sqrt[n]{a}$, the n is called the index.

Worked Example

Consider the expression $2^{\frac{3}{2}}$.

Using the Power to a Power Rule: $2^{\frac{3}{2}} = \left(2^{\frac{1}{2}}\right)^3$ or $(2^3)^{\frac{1}{2}}$.

You can use the definition of rational exponents to rewrite each expression in radical form.

$$\begin{aligned}\left(2^{\frac{1}{2}}\right)^3 &= (\sqrt{2})^3 \\ &= \sqrt{2}\cdot\sqrt{2}\cdot\sqrt{2} \\ &= 2\sqrt{2}\end{aligned}$$

$$\begin{aligned}(2^3)^{\frac{1}{2}} &= \sqrt{2^3} \\ &= \sqrt{2\cdot 2\cdot 2} \\ &= \sqrt{2^2\cdot 2} \\ &= 2\sqrt{2}\end{aligned}$$

The process of removing perfect square numbers from under a radical symbol is called extracting square roots.

You can see that $\sqrt{2}\cdot\sqrt{2}\cdot\sqrt{2} = \sqrt{2\cdot 2\cdot 2}$.

4. Use the worked example to explain the Product Property of Radicals.

The Product Property of Radicals states that $\sqrt{a}\cdot\sqrt{b} = \sqrt{a\cdot b}$ when a and b are greater than 0.