5. **Analyze the two worked examples. Use the Properties of Exponents to explain each.**

a. $\sqrt{2} \cdot \sqrt{2} = 2$

b. $\sqrt{2^2} = 2$

6. Tony and Bobby each calculate the population of bacteria when $t = \frac{5}{2}$ hours.

Tony says that when $t = \frac{5}{2}$ hours, $f\left(\frac{5}{2}\right) = (\sqrt{2})^5$ bacteria.

Bobby says that $f\left(\frac{5}{2}\right) = 4\sqrt{2}$ bacteria.

Who's correct?

Use definitions and rules to justify your reasoning. Then use the graph to estimate the value of $f\left(\frac{5}{2}\right)$ as a decimal on the graph.

In the previous activity, you looked at the exponential function $f(x) = 2^x$. When the input interval is 1, the constant ratio is 2^1, and when the input interval is $\frac{1}{2}$, the multiplier is $2^{\frac{1}{2}}$.

Now, let's think about the constant multiplier when the input interval is $\frac{1}{3}$.

$\times 2$

x	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$
$f(x)$	2^0			2^1	
	1			2	

r r r

Think about:

What is the constant multiplier you can use to build this relationship over intervals of $\frac{1}{3}$?

In this lesson, you have been writing powers with rational exponents. You have shown that you can rewrite a rational exponent in radical form, $a^{\frac{1}{n}} = \sqrt[n]{a}$.

1. Rewrite each expression as a power.

a. $\sqrt[3]{7}$

b. $\sqrt[5]{x}$

c. $\sqrt{y}$

2. Rewrite each expression in radical form.

a. $8^{\frac{1}{4}}$

b. $z^{\frac{1}{5}}$

c. $m^{\frac{1}{3}}$

3. Use the properties of exponents to rewrite $a^{\frac{m}{n}}$ in radical form.

4. Rewrite each expression in radical form.

a. $4^{\frac{3}{2}}$

b. $5^{\frac{3}{4}}$

c. $x^{\frac{4}{5}}$

d. $y^{\frac{2}{3}}$

5. Rewrite each expression as a power with a rational exponent.

a. $\left(\sqrt[4]{2}\right)^3$

b. $\left(\sqrt{5}\right)^4$

c. $\left(\sqrt[5]{x}\right)^8$

d. $\left(\sqrt[5]{y}\right)^{10}$

Let's analyze the product of radicals.

Worked Example

You can rewrite the numeric expression $(\sqrt[3]{2})^2(\sqrt{2})$ in radical form using the rules of exponents.

$(2^{\frac{1}{3}})^2(2)^{\frac{1}{2}}$	Definition of rational exponents.
$(2^{\frac{2}{3}})\ (2^{\frac{1}{2}})$	Power to a Power Rule.
$2^{\frac{2}{3}+\frac{1}{2}}$	Product Rule of Powers.
$2^{\frac{7}{6}}$	Add fractions.
$\sqrt[6]{2^7}$	Definition of rational exponents.

6. Tonya rewrote the expression $\sqrt[6]{2^7}$ in a different way.

$$2^{\frac{7}{6}} = 2^{\frac{6}{6}} \cdot 2^{\frac{1}{6}} = 2\sqrt[6]{2}$$

Is she correct? Justify your reasoning.

Let's revisit the Product Property of Radicals and the process of extracting roots.

Suppose you have the product $\sqrt{15} \cdot \sqrt{5}$. You can use properties of exponents to rewrite this radical expression.

Worked Example

$$\begin{aligned}\sqrt{15} \cdot \sqrt{5} &= 15^{\frac{1}{2}} \cdot 5^{\frac{1}{2}} \\ &= (15 \cdot 5)^{\frac{1}{2}} \\ &= (3 \cdot 5 \cdot 5)^{\frac{1}{2}} \\ &= (3 \cdot 5^2)^{\frac{1}{2}} \\ &= 3^{\frac{1}{2}} \cdot 5 \\ &= 5\sqrt{3}\end{aligned}$$

$$\begin{aligned}\sqrt{15} \cdot \sqrt{5} &= \sqrt{15 \cdot 5} \\ &= \sqrt{3 \cdot 5 \cdot 5} \\ &= \sqrt{3 \cdot 5^2} \\ &= 5\sqrt{3}\end{aligned}$$

7. Rewrite each radical expression by extracting perfect squares.

a. $\sqrt{50}$

b. $\sqrt{24}$

c. $3\sqrt{20}$

d. $\sqrt{3} \cdot \sqrt{6}$

e. $\sqrt{3} \cdot \sqrt{12}$

f. $\sqrt{8} \cdot \sqrt{12}$

9. **Consider the calculations you made throughout this lesson and the definition of a rational number to answer each question.**

   a. **Is the product of a nonzero rational number and an irrational number always, sometimes, or never a rational number? Explain your reasoning.**

   b. **Is the product of an irrational number and an irrational number always, sometimes, or never a rational number? Explain your reasoning.**

10. Rewrite each expression using the Properties of Exponents.

a. $\left(3^{\frac{3}{2}}\right)^{3}$

b. $\dfrac{\left(2^{-\frac{1}{2}}\right)^{3}}{\left(2^{\frac{1}{3}}\right)^{-1}}$

c. $\left(2x^{\frac{1}{2}}y^{\frac{1}{3}}\right)\left(3x^{\frac{1}{2}}y\right)$

d. $\left(\dfrac{24m^{\frac{3}{4}}n^{\frac{5}{2}}}{36m^{\frac{2}{7}}n^{\frac{2}{5}}}\right)^{0}$