5. Analyze the two worked examples. Use the Properties of Exponents to explain each.

a.
$$\sqrt{2} \cdot \sqrt{2} = 2$$

b.
$$\sqrt{2^2} = 2$$



6. Tony and Bobby each calculate the population of bacteria when $t = \frac{5}{2}$ hours.

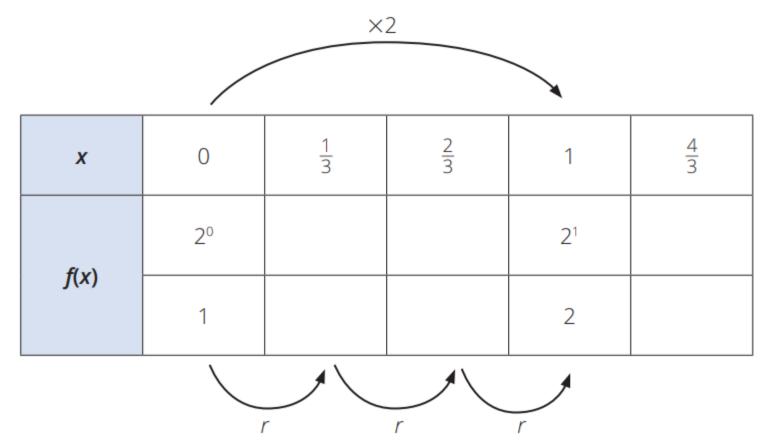
Tony says that when $t = \frac{5}{2}$ hours, $f(\frac{5}{2}) = (\sqrt{2})^5$ bacteria. Bobby says that $f(\frac{5}{2}) = 4\sqrt{2}$ bacteria.

Who's correct?

Use definitions and rules to justify your reasoning. Then use the graph to estimate the value of $f(\frac{5}{2})$ as a decimal on the graph.

In the previous activity, you looked at the exponential function $f(x) = 2^x$. When the input interval is 1, the constant ratio is 2^1 , and when the input interval is $\frac{1}{2}$, the multiplier is $2^{\frac{1}{2}}$.

Now, let's think about the constant multiplier when the input interval is $\frac{1}{3}$.





What is the constant multiplier you can use to build this relationship over intervals of $\frac{1}{3}$?

 $a^{\frac{1}{n}} = \sqrt[n]{a}$.

1. Rewrite each expression as a power.

b.
$$\sqrt[5]{x}$$

2. Rewrite each expression in radical form.

a.
$$8^{\frac{1}{4}}$$

c.
$$m^{\frac{1}{3}}$$

3. Use the properties of exponents to rewrite $a^{\frac{m}{n}}$ in radical form.

4. Rewrite each expression in radical form.

a.
$$4^{\frac{3}{2}}$$

b.
$$5^{\frac{3}{4}}$$

c.
$$x^{\frac{4}{5}}$$

d.
$$y^{\frac{2}{3}}$$

5. Rewrite each expression as a power with a rational exponent.

a.
$$(\sqrt[4]{2})^3$$

b.
$$(\sqrt{5})^4$$

d.
$$(\sqrt[5]{y})^{10}$$

Worked Example

You can rewrite the numeric expression $(\sqrt[3]{2})^2 (\sqrt{2})$ in radical form using the rules of exponents.

$$(2^{\frac{1}{3}})^2 (2)^{\frac{1}{2}}$$
 Definition of rational exponents.

$$(2^{\frac{2}{3}})$$
 $(2^{\frac{1}{2}})$ Power to a Power Rule.

$$2^{\frac{2}{3} + \frac{1}{2}}$$
 Product Rule of Powers.

$$2^{\frac{7}{6}}$$
 Add fractions.

$$\sqrt[6]{2^7}$$
 Definition of rational exponents.

6. Tonya rewrote the expression $\sqrt[6]{2^7}$ in a different way.

$$2^{\frac{7}{6}} = 2^{\frac{6}{6}} \cdot 2^{\frac{1}{6}} = 2\sqrt[6]{2}$$



Is she correct? Justify your reasoning.

Let's revisit the Product Property of Radicals and the process of extracting roots.

Suppose you have the product $\sqrt{15} \cdot \sqrt{5}$. You can use properties of exponents to rewrite this radical expression.

Worked Example

$$\sqrt{15} \cdot \sqrt{5} = 15^{\frac{1}{2}} \cdot 5^{\frac{1}{2}}$$

$$= (15 \cdot 5)^{\frac{1}{2}}$$

$$= (3 \cdot 5 \cdot 5)^{\frac{1}{2}}$$

$$= (3 \cdot 5^{\frac{1}{2}})^{\frac{1}{2}}$$

$$= (3 \cdot 5^{\frac{1}{2}})^{\frac{1}{2}}$$

$$= 3^{\frac{1}{2}} \cdot 5$$

$$= 5\sqrt{3}$$

d.
$$\sqrt{3} \cdot \sqrt{6}$$

e.
$$\sqrt{3} \cdot \sqrt{12}$$

f.
$$\sqrt{8} \cdot \sqrt{12}$$

- 9. Consider the calculations you made throughout this lesson and the definition of a rational number to answer each question.
 - a. Is the product of a nonzero rational number and an irrational number always, sometimes, or never a rational number? Explain your reasoning.

 Is the product of an irrational number and an irrational number always, sometimes, or never a rational number?
 Explain your reasoning.

a.
$$(3^{\frac{3}{2}})^3$$

b.
$$\frac{\left(2^{-\frac{1}{2}}\right)^3}{\left(2^{\frac{1}{3}}\right)^{-1}}$$

c.
$$(2x^{\frac{1}{2}}y^{\frac{1}{3}})(3x^{\frac{1}{2}}y)$$

$$d. \left(\frac{24m^{\frac{3}{4}}n^{\frac{5}{2}}}{36m^{\frac{2}{7}}n^{\frac{2}{5}}}\right)^0$$