In 2018, the population of Chicago, Illinois, was about 2.7 million, and the population of Columbus, Ohio, was about 880,000. Chicago's population had decreased from 2010 at a rate of 0.04% each year. At the same time, Columbus's population had grown at a rate of about 0.14% every year.
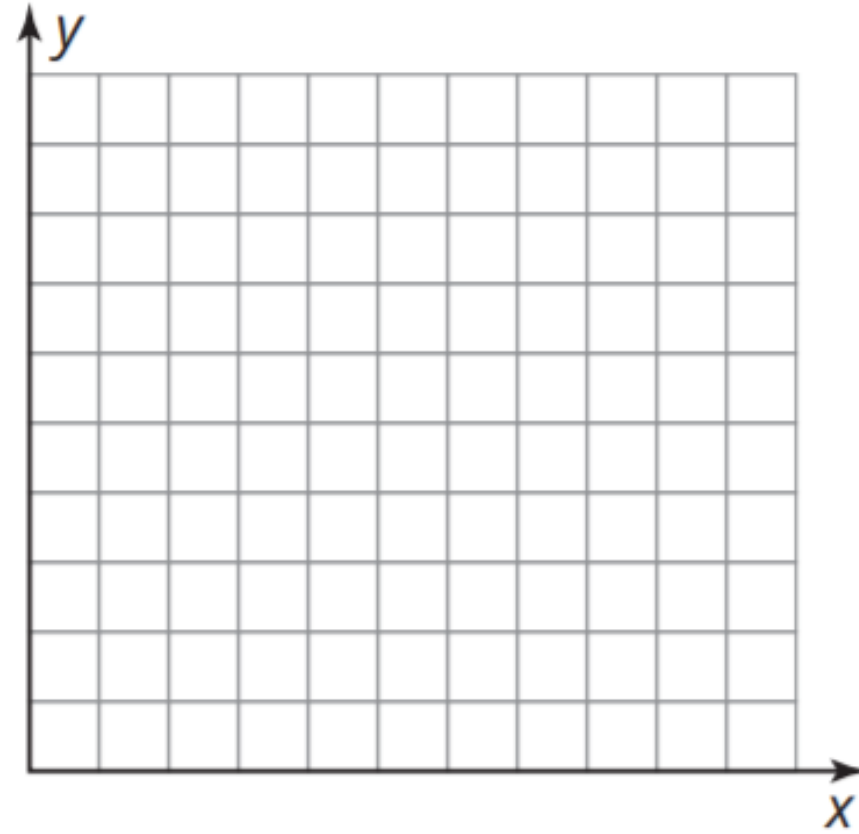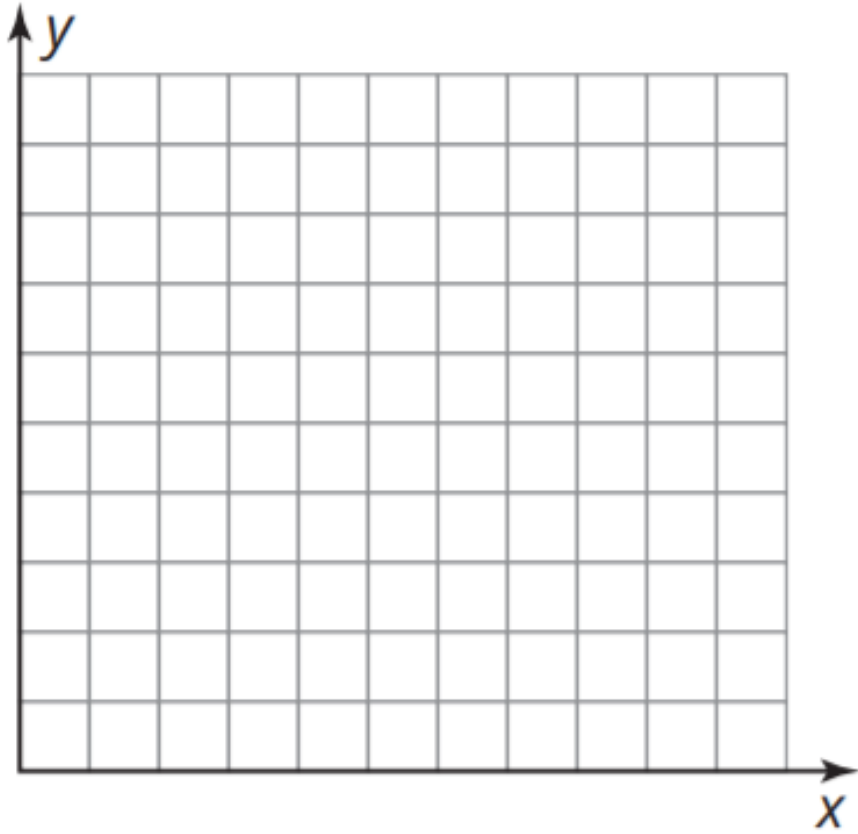
1. **Which city's population can be represented as an increasing function, and which can be represented as a decreasing function?**

Let's examine the properties of the graphs of the functions for Chicago and Columbus.

Chicago: $G(t) = 2{,}700{,}000(1 - 0.0004)^t$

Columbus: $B(t) = 880{,}000(1 + 0.0014)^t$

2. Sketch a graph of each function. Label key points.

3. The functions $G(t)$ and $B(t)$ can each be written as an exponential function of the form $f(x) = a \cdot b^x$.

a. What is the a-value for each function? What does each a-value mean in terms of this problem situation?

b. What is the b-value for each function? What does each b-value mean in terms of this problem situation?

c. Compare and explain the meanings of the expressions $(1 - 0.0004)^t$ and $(1 + 0.0014)^t$ in terms of this problem situation.

4. Analyze the *y*-intercepts of each function. Describe how you can determine the *y*-intercept of each function using only the formula for population increase or decrease.

Consider an exponential function of the form $f(x) = a \cdot b^x$ with $a > 0$. An exponential growth function has a *b*-value greater than 1 and is of the form $y = a \cdot (1 + r)^x$, where *r* is the rate of growth. The *b*-value is $1 + r$. An exponential decay function has a *b*-value greater than 0 and less than 1 and is of the form $y = a \cdot (1 - r)^x$, where *r* is the rate of decay. The *b*-value is $(1 - r)$.

A decreasing exponential function is denoted by a decimal or fractional *b*-value between 0 and 1, not by a negative *b*-value.

1. Match each situation with the appropriate function. Explain your reasoning.

Functions

$f(x) = 17{,}000 \cdot (1 - 0.015)^x$

$f(x) = 17{,}000 \cdot (1 + 0.015)^x$

$f(x) = 17{,}000 \cdot 0.975^x$

$f(x) = 17{,}000 \cdot 1.01^x$

$f(x) = 17{,}000 \cdot 1.025^x$

$f(x) = 17{,}000 \cdot 1.1^x$

1. Match each situation with the appropriate function. Explain your reasoning.

Aliso has a population of 17,000. Its population is increasing at a rate of 1.5%.

$$f(x) = 17{,}000 \cdot (1 + 0.015)^x$$

Charlestown has a population of 17,000. Its population is decreasing at a rate of 2.5%.

$$f(x) = 17{,}000 \cdot 0.975^x$$

Youngstown has a population of 17,000. Its population is decreasing at a rate of 1.5%.

$$f(x) = 17{,}000 \cdot (1 - 0.015)^x$$

Point Park has a population of 17,000. Its population is increasing at a rate of 2.5%.

$$f(x) = 17{,}000 \cdot 1.025^x$$

North Lake has a population of 17,000. Its population is increasing at a rate of 10%.

$$f(x) = 17{,}000 \cdot 1.1^x$$

Springfield has a population of 17,000. Its population is increasing at a rate of 1%.

$$f(x) = 17{,}000 \cdot 1.01^x$$

Things Are Looking Up!

In 2005, the population of a city was 42,500. By 2010, the population had grown to approximately 51,708 people.

1. **Identify any equations that are appropriate exponential models for the population of the city. Explain why. Then explain why the equations you did not choose are not appropriate models for the situation.**

$f(t) = 51{,}708(1.04)^t$

$j(t) = 51{,}708(0.96)^t$

$g(t) = 42{,}500(1.04)^{5t}$

$k(t) = 51{,}708(1.04)^{\frac{1}{5}t}$

$h(t) = 42{,}500(1.04)^t$

$m(t) = 42{,}500(0.96)^t$