The Horizontal Line and Powers

Interpreting Parameters in Context

Warm Up

Rewrite each expression using the Order of Operations.

1. $3 - (4 \cdot 5) + 6^{-1}$

2. $8 + (9^{\frac{1}{2}} - 2) \div 2$

3. $1 - \frac{5^{\frac{1}{2}}}{2} + \sqrt{5}$

Learning Goals

- Analyze equations and graphs of exponential functions.
- Match equations and graphs of exponential functions using the horizontal asymptote.
- Write and interpret exponential growth and decay functions.
- Add an exponential function and a constant function.

You have written exponential functions for problem situations. What strategies can you use to write and solve exponential equations?

Match Game

Ask yourself:

What does the form of the equation tell you about the graph?

Consider the four graphs shown.
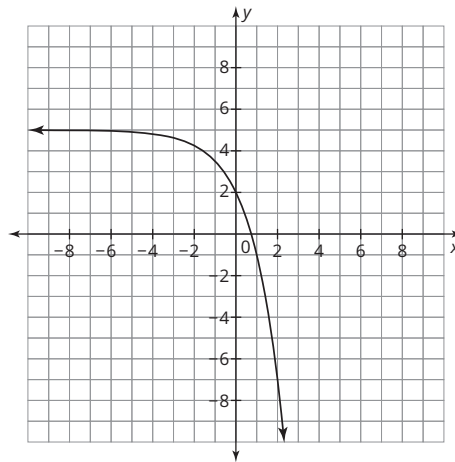
1. Match each graph with the correct function.

Equations

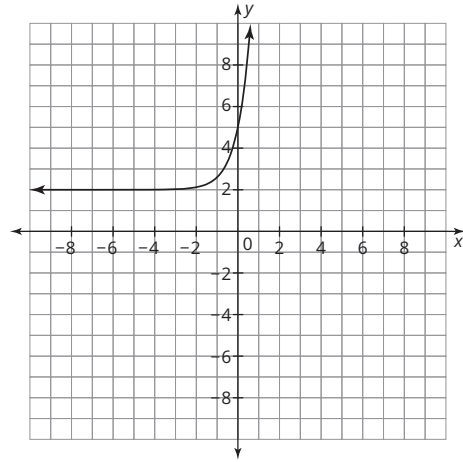$f(x) = 3(5)^x + 2$

$f(x) = 5(2)^{-x}$

$f(x) = -3(2)^x + 5$

$f(x) = -2(5)^{-x} - 3$

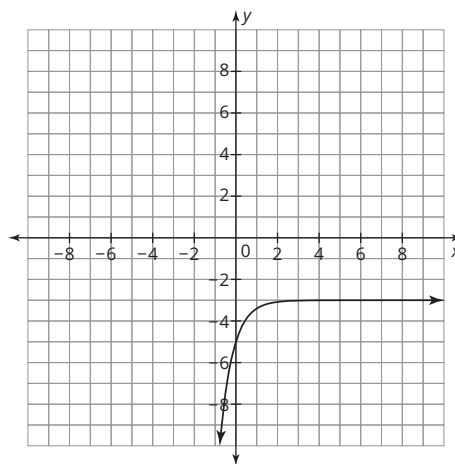**Graph 1**

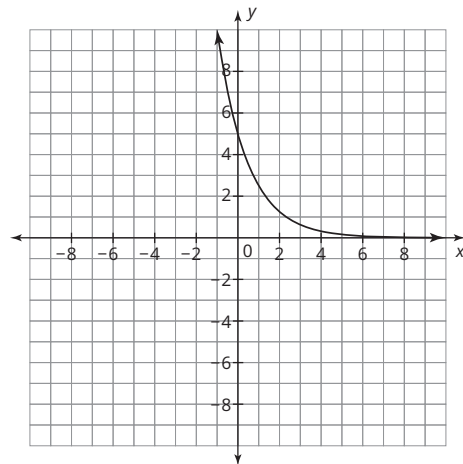**Graph 2**

Graph 3

Graph 4

2. Describe the strategies you used.

3. **Lucy and Michael disagree about the equation for Graph 2.**

Lucy 👎

The equation for Graph 2 is $f(x) = 5(2)^{-x}$.

The graph intersects the y-axis at 5.

In the form $f(x) = ab^x$, a is the y-intercept.

$f(x) = 5(2)^{-x}$ is the only equation with $a=5$.

Michael 👍

The equation for Graph 2 is $f(x) = 3(5)^x + 2$.

I know this because the graph has an asymptote of $y = 2$, and 2 is the D-value in the equation.

a. **What is the error in Lucy's thinking. Does Lucy's method sometimes work? Explain your reasoning.**

b. **What characteristic does Michael use? Will his method always work?**
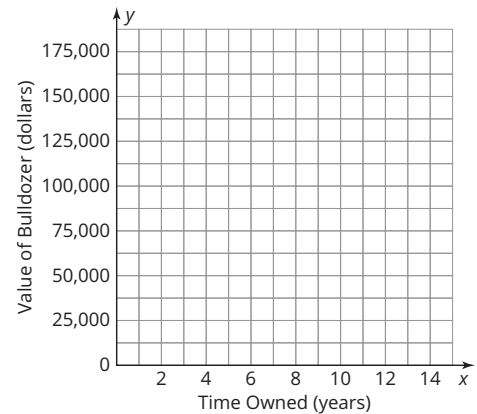
Solving Exponential Equations by Graphing

Depreciation is a decline in the value of something. Vehicles usually depreciate over time, meaning their value decreases over time. This decrease can often be represented by an exponential decay function.

A construction company bought a new bulldozer for $125,000. The bulldozer depreciates exponentially, and after 2 years, the value of the bulldozer is $80,000.

1. **Write a function to represent the value of the bulldozer as a function of the number of years it is owned. Then complete the table and graph.**

Number of Years Owned	Value of Bulldozer
0	
2.5	
5	
7	
8.5	
10	
12.5	

Value of Bulldozer (dollars) — y-axis: 0, 25,000, 50,000, 75,000, 100,000, 125,000, 150,000, 175,000

Time Owned (years) — x-axis: 2, 4, 6, 8, 10, 12, 14

Ask

yourself:

What does each point on the graph represent?

2. **The company wants to sell the bulldozer and make at least $25,000 in the sale. Use the graph to estimate the amount of time the company has to achieve this goal.**

3. **Estimate when the bulldozer will be worth:**

   a. $50,000.   **b. $10,000.**

4. **When will the bulldozer be worth $0?**

2.2 Adding Functions

Autumn has two different methods of saving her money. Analyze her situation using functions.

1. **Autumn received a graduation gift of $1000 from her wealthy aunt. She placed this money in a savings account with a 4% interest rate, compounded annually.**

   a. **Write a function $f(x)$ to model this situation. Define the variables.**

   b. **What will be the balance in Autumn's account after 5 years? 10 years? 15 years?**

   c. **Estimate when Autumn will have $1600 in her account.**

2. **Autumn also saved $500 that she keeps in a safe at home. She never touches it nor adds to it.**

   a. **Write a function $g(x)$ to model this situation. Define the variables.**

   b. **How much money will Autumn have in the safe after 5 years? 10 years? 15 years?**

   c. **When will Autumn have $1600 in the safe?**

3. **Autumn's total savings can be represented as $h(x) = f(x) + g(x)$. Write a function $h(x)$ to represent this sum and predict what the graph of $h(x)$ will look like.**

4. **Graph $f(x)$, $g(x)$, and $h(x)$ on the coordinate plane. Label each function.**
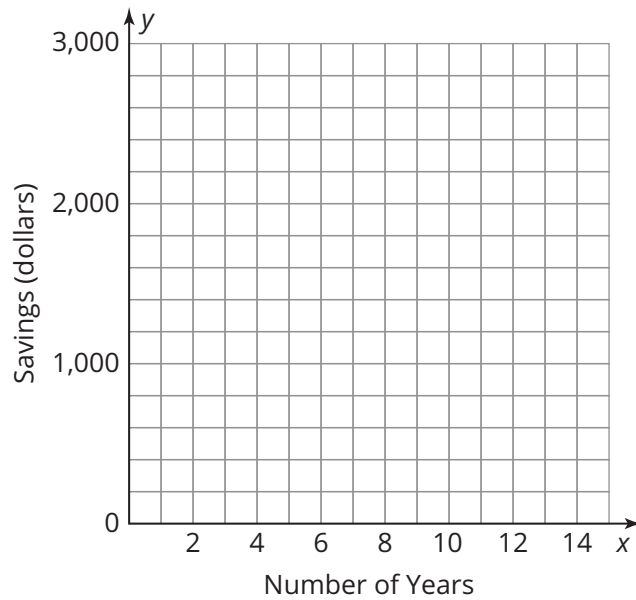
5. **Did the graph of $h(x)$ appear as you predicted? How does it relate to what you learned about transformations?**

6. **How does the graph of $h(x)$ relate to the graphs of $f(x)$ and $g(x)$?**

7. **The exponential function $h(x)$ can be written in the form $h(x) = A \cdot b^{B(x - C)} + D$. Identify three places where the value of D is evident in the graphs of the three functions.**

TALK the TALK

And More, Much More Than This...

A scientist is researching certain bacteria that have been found recently in the large animal cages at a local zoo. He starts with 200 bacteria that he intends to grow and study. He determines that every hour the number of bacteria increases by 25%.

1. **Write a function and sketch a graph to represent this problem situation. Then estimate the number of hours the scientist should let the bacteria grow to have no more than 2000 bacteria.**
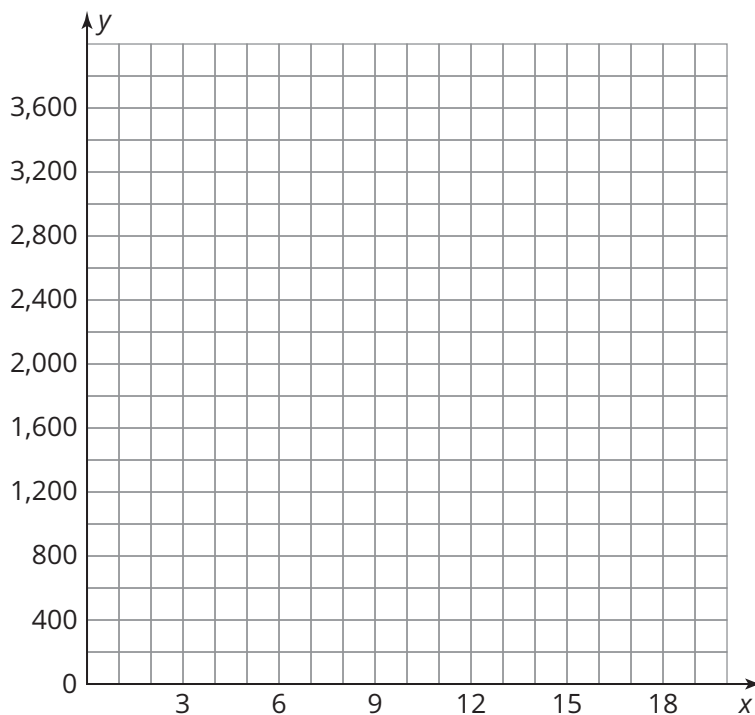
Assignment

Write

Explain how an asymptote can be identified from an exponential equation and its graph.

Remember

You can estimate the solution to an exponential equation graphically. First, graph both the exponential function and the constant function for the given y-value. Next, determine the point of intersection of the graph of the exponential function and the horizontal line. Lastly, identify the x-value of the coordinate pair as the solution.

Practice

1. Ryan bought a brand new car for $18,000. Its value depreciated at a rate of 1.2%.
   a. Write a function to represent the value of the car as a function of time. Use technology to estimate the number of years it will take for the value to reach each given amount.
   b. $17,000
   c. $15,000
   d. half of the starting value
   e. one-third the starting value
   f. $0
   g. $10,000

2. Oscar wants to own a bee colony so that he can extract honey from the hive. He starts a colony with 5,000 bees. The number of bees grows exponentially with a growth factor of 12% each month.
   a. Write a function, $f(x)$, for the bee population that can be used to determine the number of bees in the colony, based on the month, x.
   b. Use technology to graph the function, $f(x)$.
   c. Oscar feels that in order to get a decent amount of honey, there should be at least 15,000 bees in the colony. Estimate how many months it will take Oscar until he has 15,000 bees.

Stretch

Julissa and Megan developed a new art app for smart phones. The table shows the number of customers who downloaded the app by month.

1. Julissa thinks that the equation that represents the data in the table is $y = 4(2)^x$. Determine if Julissa is correct. Explain your reasoning.
2. Determine a different exponential equation that represents the data in the table. Use the equation $y = a \cdot b^{f(x)}$, where $f(x)$ is a function of x and $a = 2$.

Month	Number of Downloads
0	4
1	8
2	16
3	32
4	64
5	128

Review

1. Eleanor received $1500 for her birthday. She is going to spend $500 and wants to put the rest into an account that will earn interest. She is considering two different accounts. Account A earns 6.5% annual simple interest. Account B earns 4.5% annual compound interest.

   a. Write a function for each account that can be used to determine the balance in the account based on the year, t.

   b. Graph the functions for Accounts A and B using technology. Then, graph the functions. Be sure to label your graph.

   c. If Eleanor plans on leaving the money in the account for 12 years, which account should she use to deposit her money? Explain your reasoning.

   d. If Eleanor plans on leaving the money in the account for 25 years, which account should she use to deposit her money? Explain your reasoning.

2. Solve each equation for x using common bases.

   a. $2^{x+1} = 256$

   b. $4^3 = 2^x$