Warm Up

Determine each product or quotient. Write each answer with the given base and a single exponent or in radical form.

1. $2^{\frac{1}{2}} \cdot 2^3$

2. $\frac{4^{\frac{1}{4}}}{4}$

3. $(a^2)^{\frac{1}{3}} \cdot (a^3)^{\frac{1}{9}}$

4. $\frac{(x^{\frac{2}{5}})^2}{x}$

Music of the Night

Recall that the transformational form of a function $f(x)$ is given by:

$$g(x) = Af(B(x - C)) + D$$

1. Write the basic exponential function, $f(x) = 2^x$, in transformational form. Use integer values to represent *A*, *B*, *C*, and *D*.

2. What equation represents the location of the horizontal asymptote of a general exponential function? Explain your answer.

A horizontal asymptote is a horizontal line that a function gets closer and closer to, but never intersects.

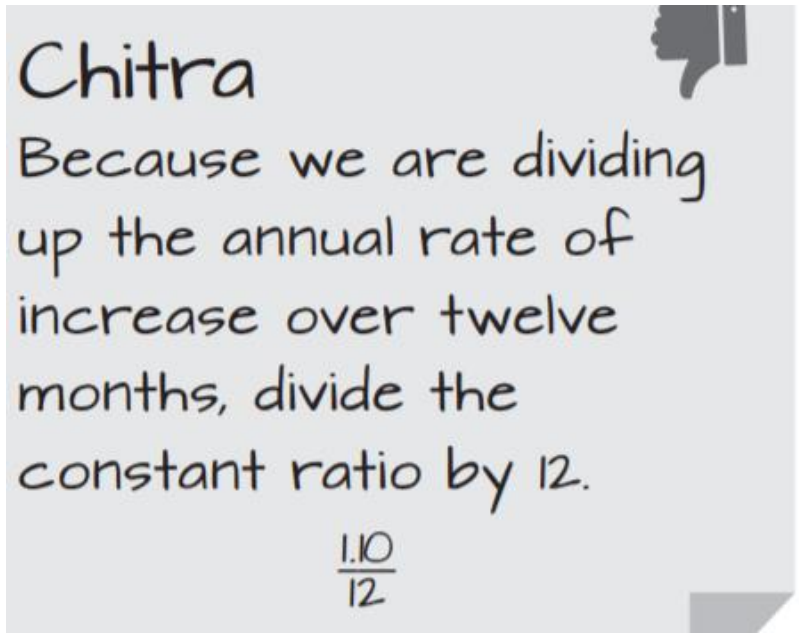
In an exponential function, the *B*-value is the coefficient of the exponent. Let's use the power of exponents to investigate the *B*-value of exponential functions.

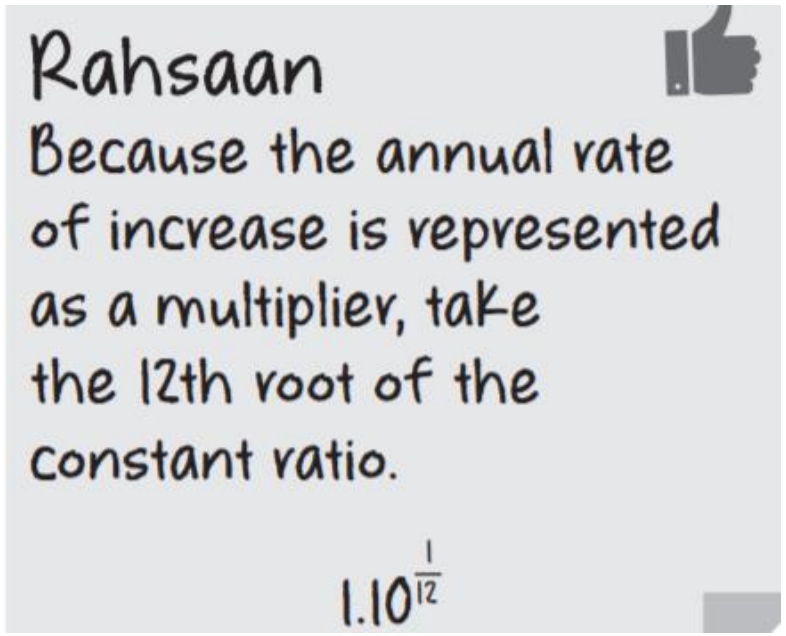
Simone has invested $500 in a mutual fund which has shown an annual increase of about 10%.

1. **Write a function, $f(t)$, that represents Simone's investment in terms of t, time in years.**

Suppose Simone is interested in determining the monthly rate of increase. What is the approximate equivalent monthly rate of increase for her mutual fund?

2. Consider the responses from two of Simone's friends. Describe the differences in their reasoning and why Rahsaan is correct.





Kirk wants to write a function that is equivalent to the annual rate of increase but reveals the monthly rate of increase.

3. Explain why Kirk's reasoning is not correct.

Kirk

Since Simone's monthly rate of increase is the twelfth root of the annual rate of increase, I can use the function

$f(x) = 500 \cdot \left(1.10^{\frac{1}{12}}\right)^{t}$.

To rewrite the function representing Simone's annual increase as an equivalent function that reflects the monthly rate of increase, you must change the *B*-value. The *B*-value of an exponential function can be written as the coefficient of *x*.

$$f(x) = a \cdot (b)^{Bx}$$

You can use what you know about common bases to rewrite the expression in an equivalent form.

$$\left(1.10^{\frac{1}{12}}\right)^{Bx} = (1.10)^{x}$$

$$(1.10)^{\frac{Bx}{12}} = (1.10)^{x} \quad \text{Apply the Power to a Power Rule.}$$

$$\frac{Bx}{12} = x \quad \text{The bases are the same, so the exponents must be equivalent expressions.}$$

$$Bx = 12x \quad \text{Multiply both sides by 12.}$$

$$B = 12$$

So, the function $f(x) = 500 \cdot \left(1.10^{\frac{1}{12}}\right)^{12x}$ is equivalent to the function $f(x) = 500 \cdot (1.10)^{x}$.

4. **Suppose Simone wants to determine how much her mutual fund increases each quarter. Rewrite the original function in an equivalent form that reveals the approximate equivalent quarterly rate of increase.**

5. **What is Simone's monthly increase, as a percent?**

ACTIVITY
3.2
Horizontal Dilations of Exponential Functions

You have studied the effects of the *A*-, *C*-, and *D*-values on the graphs of functions. Now let's consider how the *B*-value affects the graph of a function.

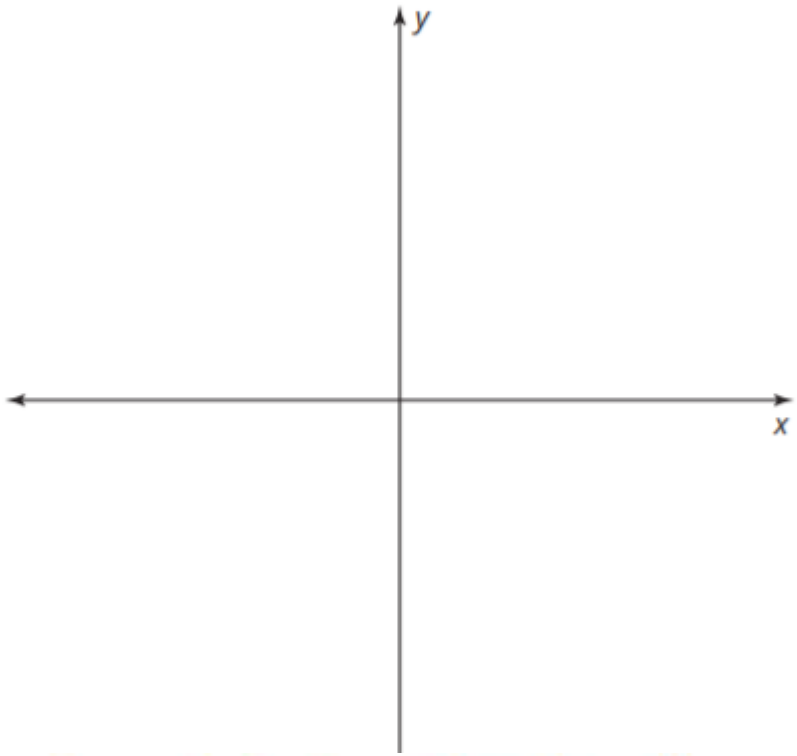
Consider the exponential functions, where $h(x) = 2^x$ is the basic function.

- $w(x) = 2^{\frac{1}{2}x}$
- $z(x) = 2^{2x}$

1. Write the functions $w(x)$ and $z(x)$ in terms of $h(x)$.

2. Use technology to sketch and label the graph of each function.

Label key points on graphs of functions, such as the *y*-intercept and the horizontal asymptote.



3. Compare the graphs of $w(x)$ and $z(x)$ to the graph of the basic function $h(x)$. What do you notice?

4. Write the *x*-value of each of the corresponding reference points on $w(x)$ and $z(x)$.

$h(x) = 2^x$	$w(x) = 2^{(\frac{1}{2}x)}$	$z(x) = 2^{(2x)}$
$\left(-2, \frac{1}{4}\right)$	$\left(\_\_\_\_, \frac{1}{4}\right)$	$\left(\_\_\_\_, \frac{1}{4}\right)$
$\left(-1, \frac{1}{2}\right)$	$\left(\_\_\_\_, \frac{1}{2}\right)$	$\left(\_\_\_\_, \frac{1}{2}\right)$
(0, 1)	(____, 1)	(____, 1)
(1, 2)	(____, 2)	(____, 2)
(2, 4)	(____, 4)	(____, 4)