Warm Up

Use the properties of logarithms to rewrite each expression.

1. $\log_3 (8x^4)$

2. $\log_2 \left(\frac{5y^6}{x^4}\right)$

3. $2 \log x - 2 \log y$

$$t = \frac{\log\left(\frac{C}{A}\right)}{\log(1 - r)}$$

where t is the time in hours since the medicine was administered, C is the current amount of medicine remaining in the patient's body in milligrams, A is the original dose of the medicine in milligrams, and r is the rate at which the medicine is metabolized.

1. **A patient is given 10 milligrams of medicine which is metabolized at the rate of 20% per hour. How long will it take for 2 milligrams of the medicine to metabolize?**

Worked Example

$$4 = \frac{\log\left(\frac{9}{12}\right)}{\log(1 - r)}$$ Substitute the values for t, C, and A into the formula.

$4 \log(1 - r) = \log(0.75)$ Multiply both sides of the equation by $\log(1 - r)$.

$4 \log(1 - r) \approx -0.125$ Evaluate $\log(0.75)$.

$\log(1 - r) \approx -0.03125$ Divide both sides of the equation by 4.

$10^{-0.03125} \approx 1 - r$ Rewrite as an exponential equation.

$r \approx 1 - 10^{-0.03125}$ Isolate the variable r.

$r \approx 0.0694$

The medicine is metabolized at an approximate rate of 6.94% per hour.

2. **Six hours after administering a 20-milligram dose of medicine, 5 milligrams remain in a patient's body. At what rate is the medicine metabolized?**

	Example	First rewrite as an exponential equation. Then solve for x.	First apply the Change of Base Formula. Then solve for x.
Argument Is Unknown	$\log_5 x = 3.1$		
Exponent Is Unknown	$\log_8 145 = x$		
Base Is Unknown	$\log_x 24 = 6.7$		

5. **Circle the logarithmic equations that can be solved more efficiently when rewritten as exponential equations. Draw a box around the equations that can be solved more efficiently by applying the Change of Base Formula. Explain your choice.**

a. $\log_4 (x + 3) = \frac{1}{2}$

b. $\log_{4.5} 9 = x - 1$

c. $\log_{x+2} 7.1 = 3$

d. $\log_3 4.6 = 2 - x$

e. $\ln (x + 4) = 3.8$

f. $\log_{11} 12 = x - 7$

g. $\log_{1-x} 8 = 14.7$

h. $\log (4 - x) = 1.3$

6. Solve each logarithmic equation. Check your work.

a. $\log_2 (x^2 - 6x) = 4$

b. $\log_6 (x^2 + x) = 1$

1. Analyze Georgia's, Santiago's, and Lorenzo's work.

Georgia

$$\log 5 + \log x = 2$$
$$\log (5 + x) = 2$$
$$10^2 = 5 + x$$
$$95 = x$$

Santiago

$$\log 5 + \log x = 2$$
$$5x = 2$$
$$x = \frac{2}{5}$$

Lorenzo

$$\log 5 + \log x = 2$$
$$5 + x = 2$$
$$x = -3$$

b. Solve $\log 5 + \log x = 2$. Check your work.

2. Solve each logarithmic equation. Check your work.

a. $\log_5 45x - \log_5 3 = 1$

b. $\log_2 8 + 3 \log_2 x = 6$

c. $\ln 18x - \ln 6 = 2$

3. **Pippa and Kate disagree about the solution to the logarithmic equation $\log_5 x^2 - \log_5 4 = 2$.**

$$\log_5 x^2 - \log_5 4 = 2$$

$$\log_5 \left(\frac{x^2}{4}\right) = 2$$

$$5^2 = \frac{x^2}{4}$$

$$25 = \frac{x^2}{4}$$

$$100 = x^2$$

$$x = 10, -10$$

Kate says the solutions are $x = 10$, $x = -10$. Pippa says that the solution $x = -10$ should be rejected because the argument of a logarithm must be greater than zero.

Who is correct? Explain your reasoning.

4. Solve $\log_3 (x - 4) + \log_3 (x + 2) = 3$. Check your work.

4. Solve $\log_3 (x - 4) + \log_3 (x + 2) = 3$. Check your work.

Elijah

$2 \log 6 = \log x - \log 2$

$\log (6^2) = \log \left(\frac{x}{2}\right)$

$36 = \frac{x}{2}$

$72 = x$

Check:

$2 \log 6 \stackrel{?}{=} \log 72 - \log 2$

$\log (6^2) \stackrel{?}{=} \log \left(\frac{72}{2}\right)$

$\log 36 = \log 36$

Zander

$2 \log 6 = \log x - \log 2$

$\log (6^2) - \log x + \log 2 = 0$

$\log \left(\frac{36}{x}\right) + \log 2 = 0$

$\log \left(\frac{72}{x}\right) = 0$

$\frac{72}{x} = 10^0$

$\frac{72}{x} = 1$

$72 = x$

Check:

$2 \log 6 \stackrel{?}{=} \log 72 - \log 2$

$\log (6^2) \stackrel{?}{=} \log \left(\frac{72}{2}\right)$

$\log 36 = \log 36$