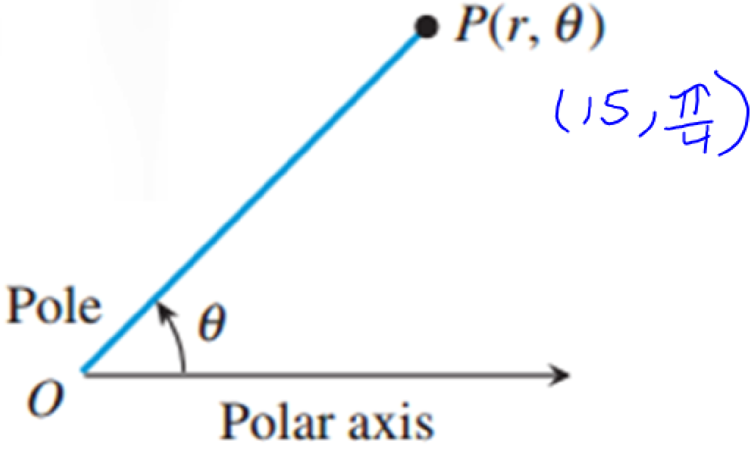
Polar Coordinates

Polar Coordinate System

A **polar coordinate system** is a plane with a point O, the **pole**, and a ray from O, the **polar axis**, as shown in Figure 6.35. Each point P in the plane is assigned as **polar coordinates** follows: r is the **directed distance** from O to P, and θ is the **directed angle** whose initial side is on the polar axis and whose terminal side is on the line OP.

As in trigonometry, we measure θ as positive when moving counterclockwise and negative when moving clockwise. If $r > 0$, then P is on the terminal side of θ. If $r < 0$, then P is on the terminal side of $\theta + \pi$. We can use radian or degree measure for the angle θ as illustrated in Example 1.

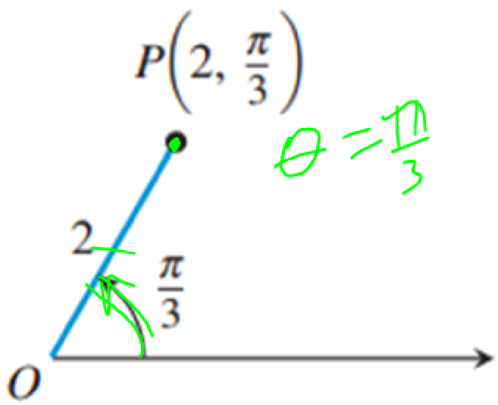


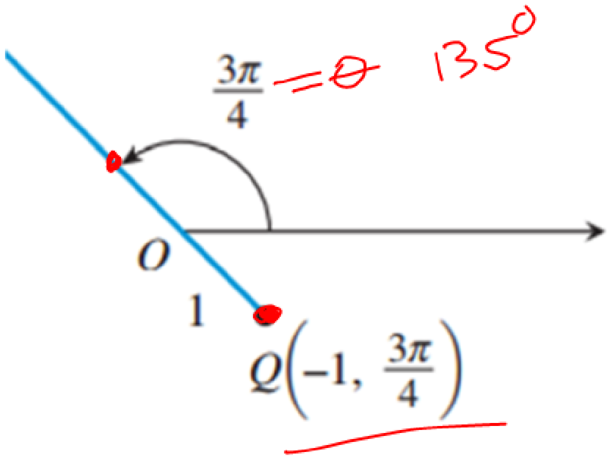
EXAMPLE 1 Plotting Points in the Polar Coordinate System

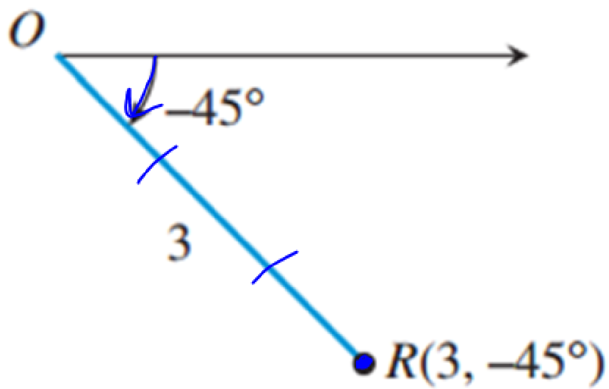
Plot the points with the given polar coordinates.

(a) $P(2, \pi/3)$ **(b)** $Q(-1, 3\pi/4)$ **(c)** $R(3, -45°)$

$r=2$ $r=-1$ $r=3$ $\theta=-45°$

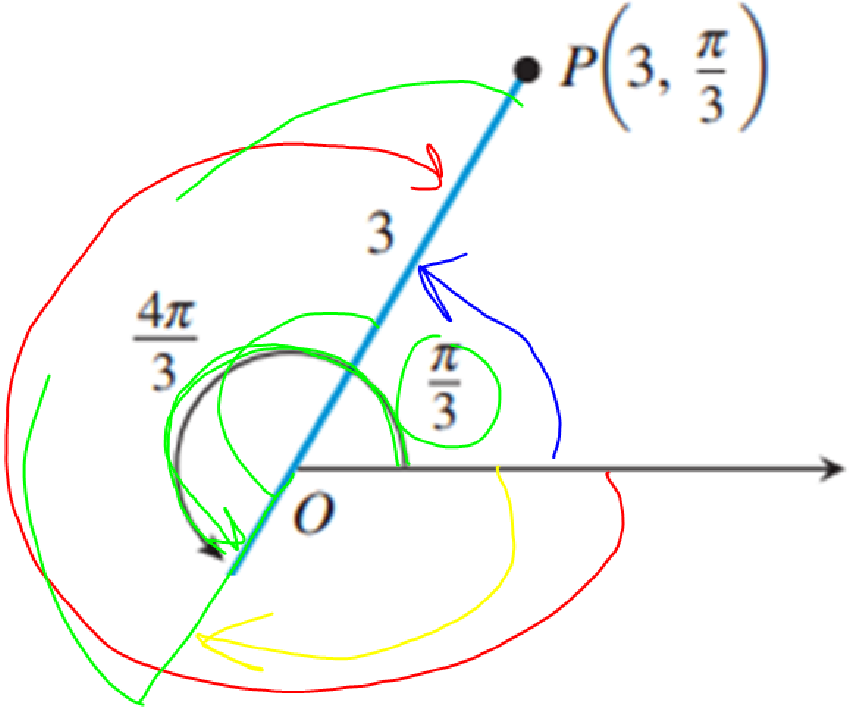






EXAMPLE 2 Finding all Polar Coordinates for a Point

If the point P has polar coordinates $(3, \pi/3)$, find all polar coordinates for P.



$P\left(3, -\frac{5\pi}{3}\right)$

$P\left(-3, \frac{4\pi}{3}\right)$

$P\left(-3, -\frac{2\pi}{3}\right)$

EXAMPLE 3 Converting from Polar to Rectangular Coordinates

Find the rectangular coordinates of the points with the given polar coordinates.

(x, y)

(a) $P(3, 5\pi/6)$ **(b)** $Q(2, -200°)$

$x = 3 \cos \frac{5\pi}{6}$

$x = 3\left(-\frac{\sqrt{3}}{2}\right)$

$x = -\frac{3\sqrt{3}}{2}$

$y = 3 \sin \frac{5\pi}{6}$

$= 3\left(\frac{1}{2}\right) = \frac{3}{2}$

$P\left(-\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$

$P(-2.6, 1.5)$

$x = 2 \cos(-200°) = -1.88$

$y = 2 \sin(-200°) = .68$

$Q(-1.88, .68)$

EXAMPLE 4 Converting from Rectangular to Polar Coordinates

Find two polar coordinate pairs for the points with given rectangular coordinates.

(a) $P(-1, 1)$ **(b)** $Q(-3, 0)$

(x, y)

$$r^2 = x^2 + y^2 \qquad \tan\theta = \frac{y}{x}$$

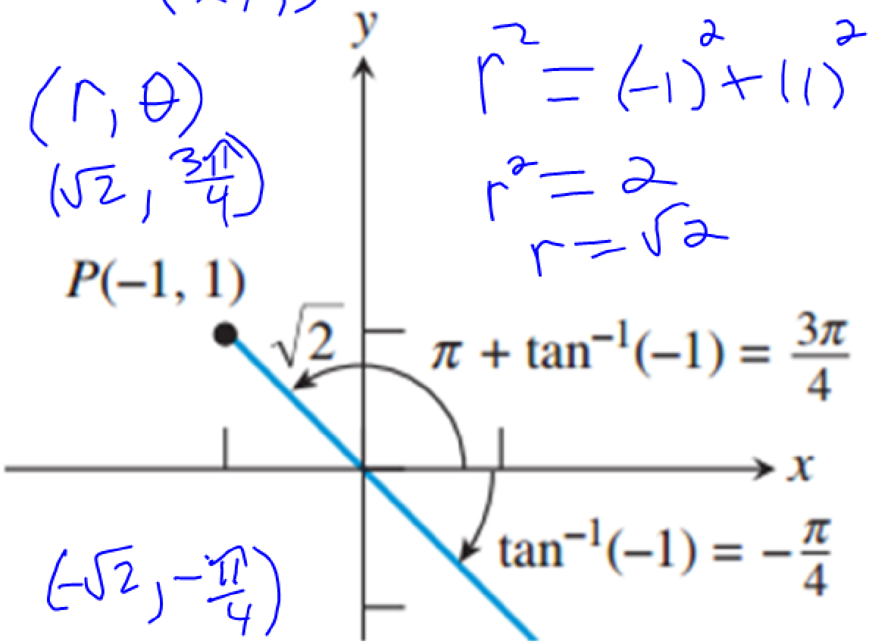
(r, θ)

$\left(\sqrt{2}, \frac{3\pi}{4}\right)$

$r^2 = (-1)^2 + (1)^2$

$r^2 = 2$

$r = \sqrt{2}$

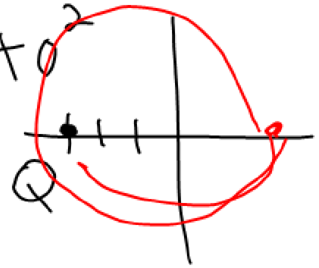


$\left(-\sqrt{2}, -\frac{\pi}{4}\right)$

$r^2 = (-3)^2 + 0^2$

$r^2 = 9$

$r = 3$



$r = 3$

(r, θ)

$(3, \pi)$ $(-3, 2\pi)$

$(3, -\pi)$ $(-3, -2\pi)$

$(-3,$