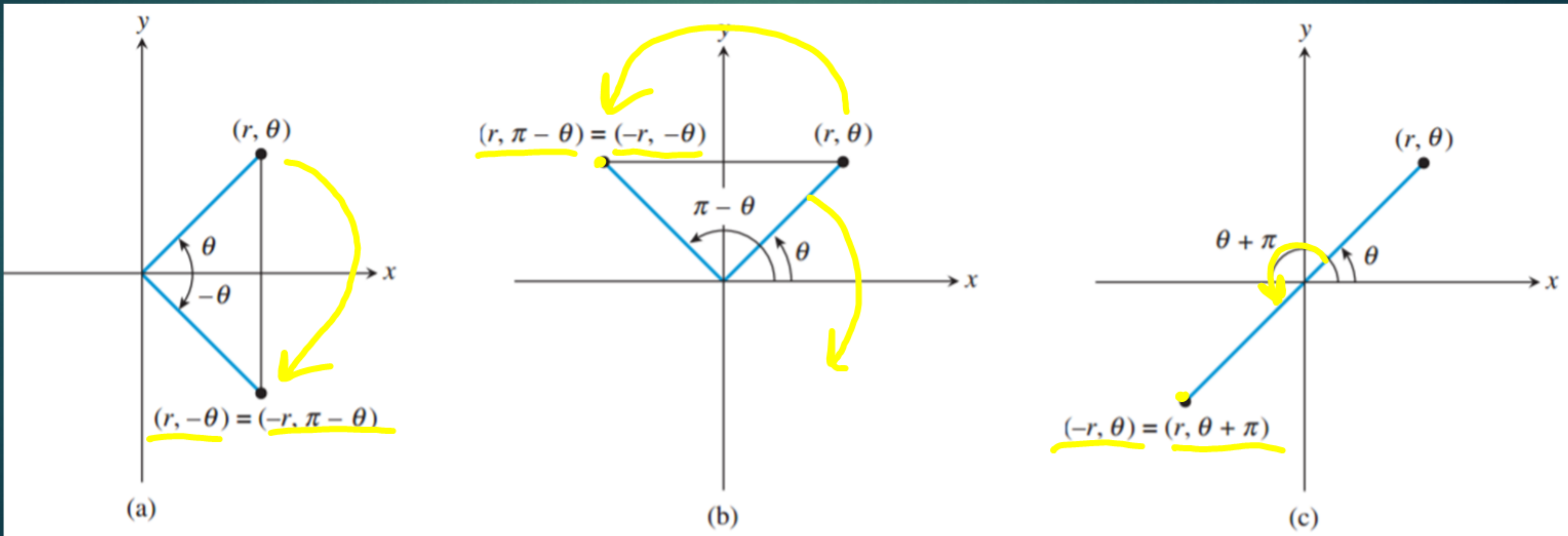
Graphs of Polar Equations

The three types of symmetry figures to be considered will have are:

1. The x-axis (polar axis) as a line of symmetry (Figure 6.45a).
2. The y-axis (the line $\theta = \pi/2$) as a line of symmetry (Figure 6.45b).
3. The origin (the pole) as a point of symmetry (Figure 6.45c).



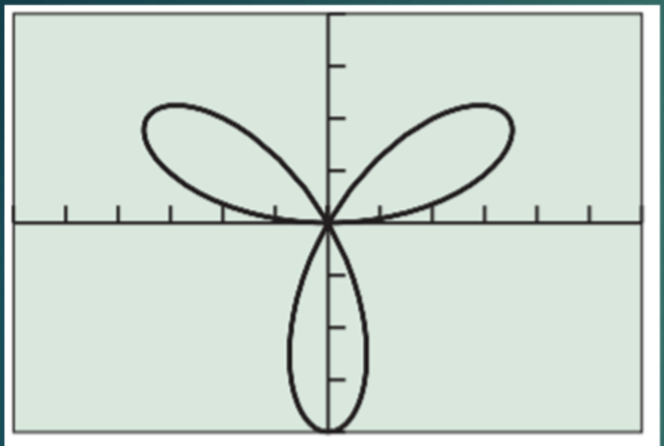
Symmetry Tests for Polar Graphs

The graph of a polar equation has the indicated symmetry if either replacement produces an equivalent polar equation.

To Test for Symmetry	Replace	By
1. about the x-axis,	(r, θ)	$(r, -\theta)$ or $(-r, \pi - \theta)$.
2. about the y-axis,	(r, θ)	$(-r, -\theta)$ or $(r, \pi - \theta)$.
3. about the origin,	(r, θ)	$(-r, \theta)$ or $(r, \theta + \pi)$.

EXAMPLE 1 Testing for Symmetry

Use the symmetry tests to prove that the graph of $r = 4 \sin 3\theta$ is symmetric about the y-axis.



Test for y-axis symmetry

$(-r, -\theta) \leftarrow$

$-r = 4\sin(-3\theta)$

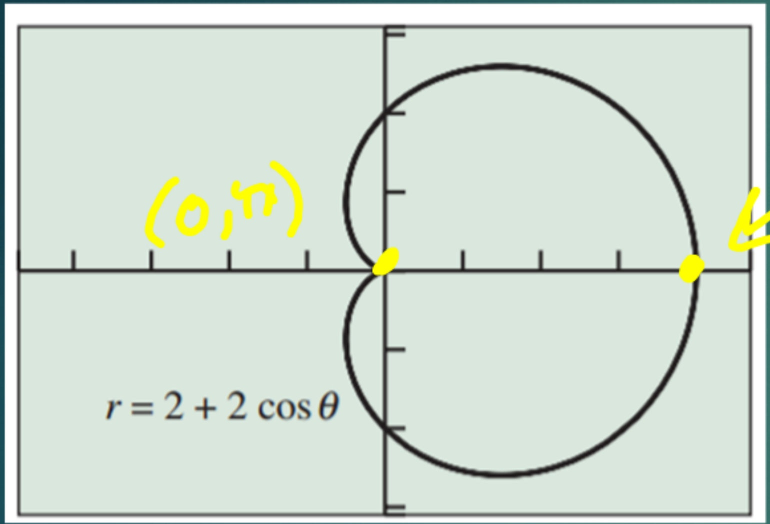
$-r = -4\sin(3\theta)$

$r = 4\sin(3\theta)$

makes the original!

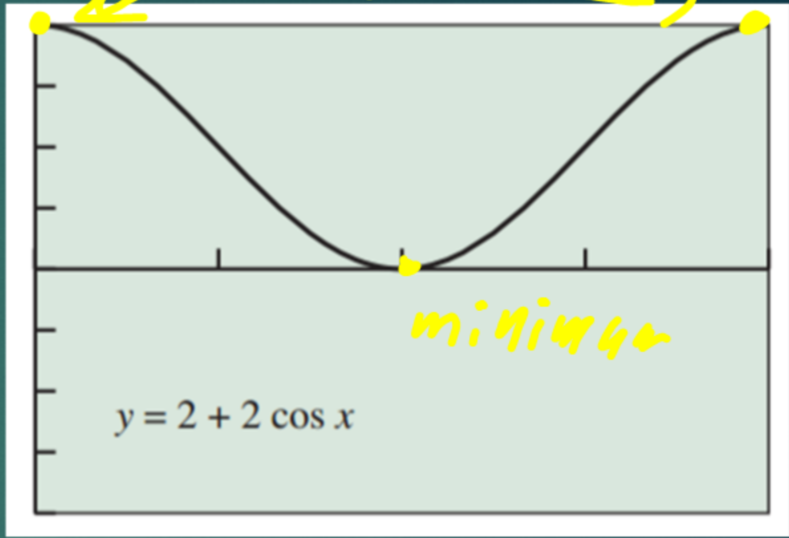
EXAMPLE 2 Finding Maximum r-Values

Find the maximum r-value of $r = 2 + 2\cos\theta$.



(4,0)
(4,2π)

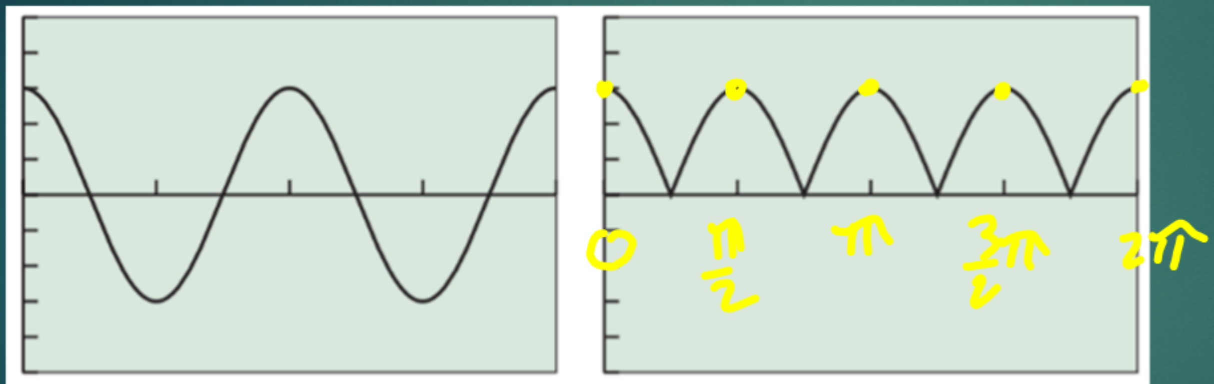
maximum

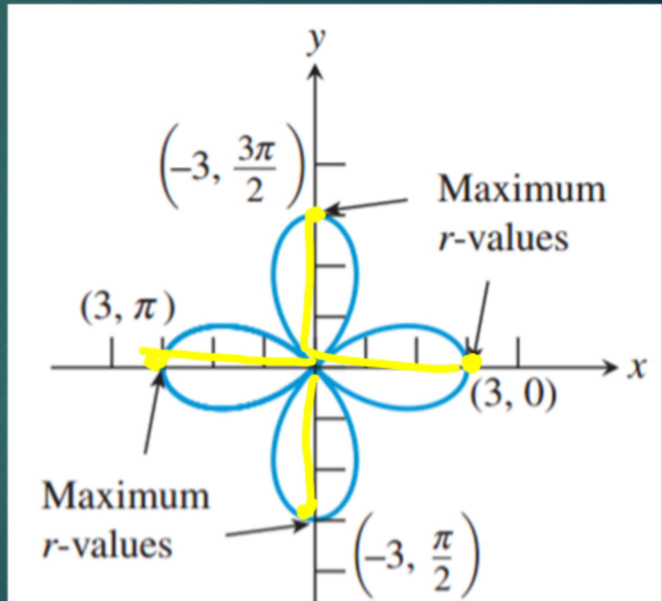


EXAMPLE 3 Finding Maximum *r*-Values

Identify the points on the graph of $r = 3\cos 2\theta$ for $0 \le \theta \le 2\pi$ that give maximum r-values.

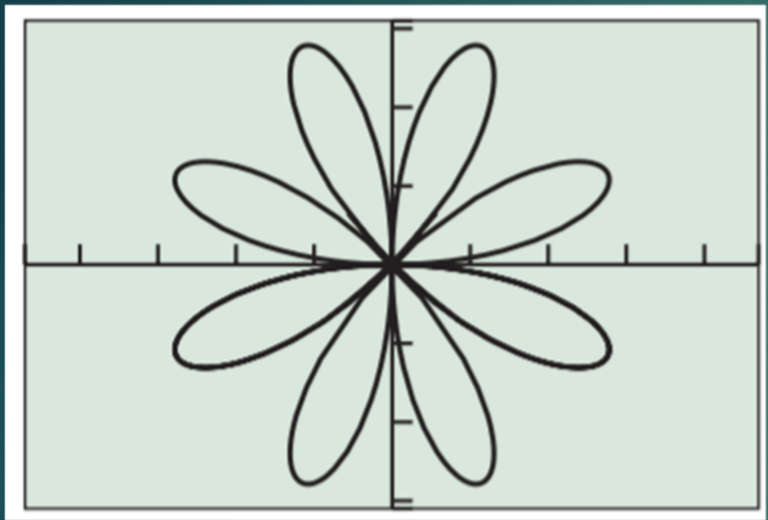
$y = |3\cos 2\theta|$





EXAMPLE 4 Analyzing a Rose Curve

Analyze the graph of the rose curve $r = 3 \sin 4\theta$.



Domain: $(-\infty, \infty)$

Range: $[-3, 3]$

Continuous

Symmetry: x-axis, y-axis, origin

Bounded

NO Asymptotes

Graphs of Rose Curves

The graphs of $r = a \cos n\theta$ and $r = a \sin n\theta$, where $n > 1$ is an integer, have the following characteristics:

Domain: All reals
Range: $[-|a|, |a|]$
Continuous
Symmetry: n even, symmetric about x-, y-axis, origin
n odd, $r = a \cos n\theta$ symmetric about x-axis
n odd, $r = a \sin n\theta$ symmetric about y-axis
Bounded
Maximum r-value: $|a|$
No asymptotes
Number of petals: n, if n is odd
$2n$, if n is even

Graphs of Rose Curves

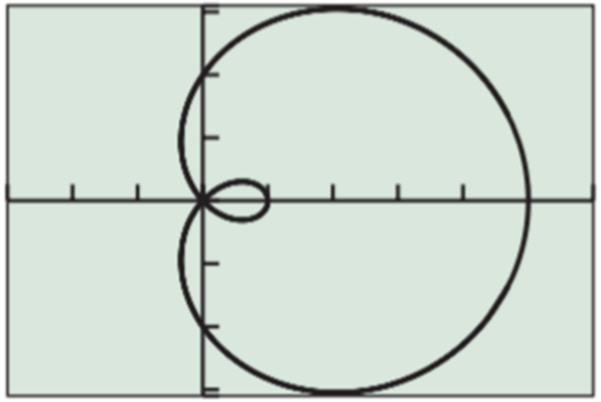
The graphs of $r = a \cos n\theta$ and $r = a \sin n\theta$, where $n > 1$ is an integer, have the following characteristics:

Domain: All reals
Range: $[-|a|, |a|]$
Continuous
Symmetry: n even, symmetric about x-, y-axis, origin
n odd, $r = a \cos n\theta$ symmetric about x-axis
n odd, $r = a \sin n\theta$ symmetric about y-axis
Bounded
Maximum r-value: $|a|$
No asymptotes
Number of petals: n, if n is odd
$2n$, if n is even

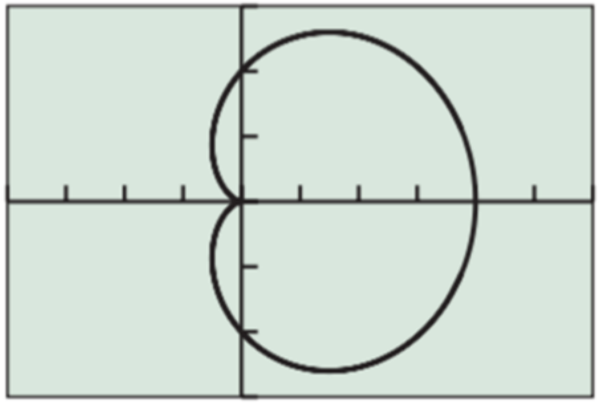
Limaçon Curves

The **limaçon curves** are graphs of polar equations of the form

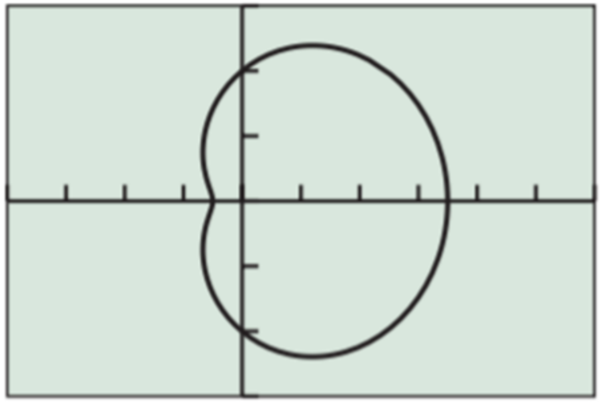
$$r = a \pm b \sin \theta \quad \text{and} \quad r = a \pm b \cos \theta$$



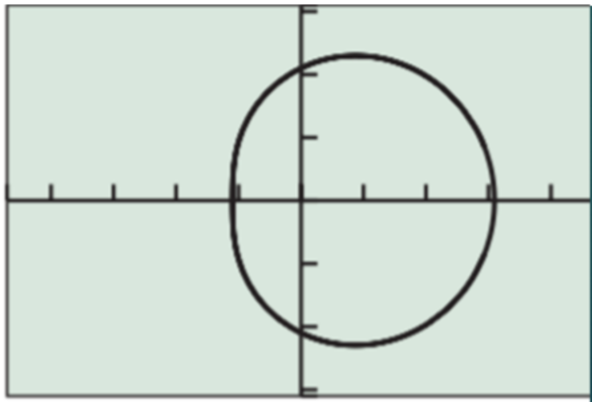
Limaçon with an inner loop: $\frac{a}{b} < 1$



Cardioid: $\frac{a}{b} = 1$



Dimpled limaçon: $1 < \frac{a}{b} < 2$



Convex limaçon: $\frac{a}{b} \geq 2$