

2. Given  $z^2 + 2z - 15 = (z - 3)(z + 5)$ , write another polynomial in general form that has a factored form of  $(z - 3)(z + 5)$  with different values for  $z$ .

A special form of a polynomial is a perfect square trinomial. A perfect square trinomial has first and last terms that are perfect squares and a middle term that is equivalent to 2 times the product of the first and last term's square root.

Factoring a perfect square trinomial can occur in two forms.

$$a^2 - 2ab + b^2 = (a - b)^2$$

$$a^2 + 2ab + b^2 = (a + b)^2$$

**3. Determine which of the polynomial expression(s) is a perfect square trinomial and write it as the square of a sum or difference. If it is not a perfect square trinomial, explain why not.**

**a.  $x^4 + 14x^2 - 49$**

**b.  $16x^2 - 40x + 100$**

c.  $64x^2 - 32x + 4$

d.  $9x^4 + 6x^2 + 1$

In polynomials of 4 terms, you may notice that although not all terms share a common factor, pairs of terms might share a common factor. In this situation, you can factor by grouping.

M2-16

**4. Colt factors the polynomial expression  $x^3 + 3x^2 - x - 3$ .**

Colt



$$x^3 + 3x^2 - x - 3$$

$$x^2(x + 3) - 1(x + 3)$$

$$(x + 3)(x^2 - 1)$$

$$(x + 3)(x + 1)(x - 1)$$

- a. Explain the steps Colt took to factor the polynomial expression.

M2-16

$$x^3 + 3x^2 - x - 3$$

$$x^2(x + 3) - 1(x + 3) \quad \text{Step 1:}$$

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$$(x + 3)(x^2 - 1) \quad \text{Step 2:}$$

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$$(x + 3)(x + 1)(x - 1) \quad \text{Step 3:}$$

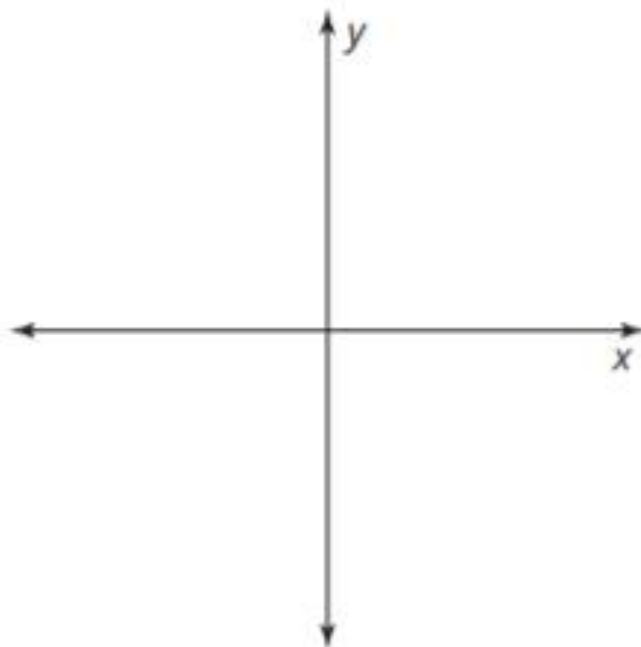
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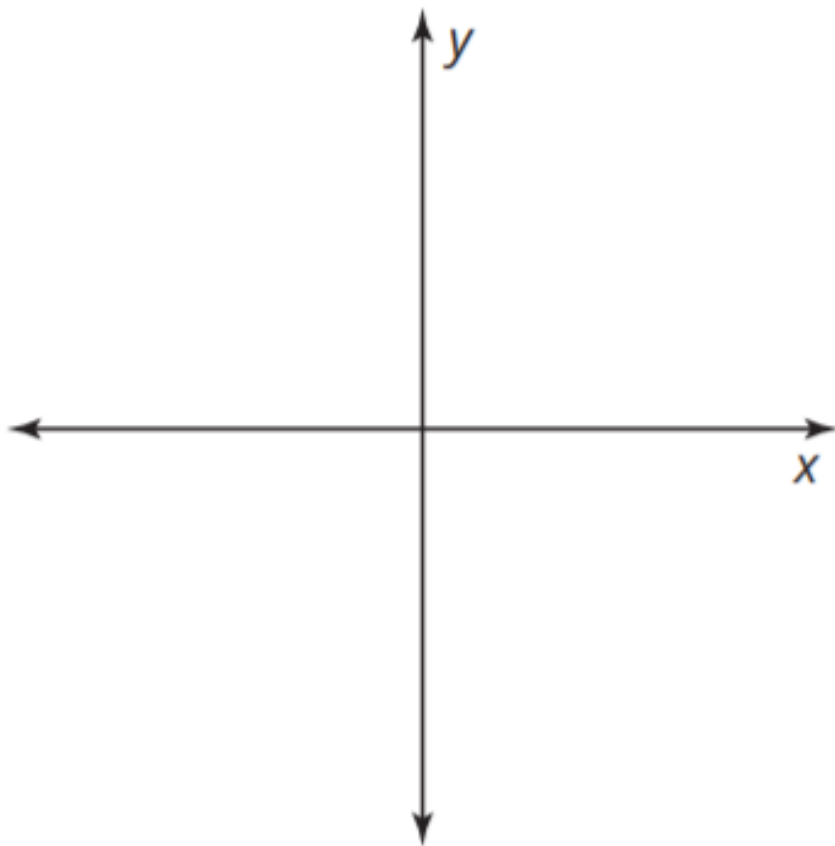
b. Use the factors to identify the zeros of  $f(x) = x^3 + 3x^2 - x - 3$  and then sketch the graph.

M2-16



5. Use factor by grouping to factor and identify the zeros of  $f(x) = x^3 + 7x^2 - 4x - 28$ . Then sketch the polynomial.

M2-17



6. Braxton and Kenny both factor the polynomial expression  $x^3 + 2x^2 + 4x + 8$ . Analyze the set of factors in each student's work. Describe the set of numbers over which each student factored.

Braxton



$$x^3 + 2x^2 + 4x + 8$$

$$x^2(x + 2) + 4(x + 2)$$

$$(x^2 + 4)(x + 2)$$

Kenny



$$x^3 + 2x^2 + 4x + 8$$

$$x^2(x + 2) + 4(x + 2)$$

$$(x^2 + 4)(x + 2)$$

$$(x + 2i)(x - 2i)(x + 2)$$



## Worked Example

Factor  $x^4 - 29x^2 + 100$  using quadratic form.

$$x^4 - 29x^2 + 100$$

Determine whether you can factor the given trinomial into 2 factors.

$$(x^2 - 4)(x^2 - 25)$$

Determine whether you can continue to factor each binomial.

$$(x - 2)(x + 2)(x - 5)(x + 5)$$

7. Factor each polynomial over the set of complex numbers. Use the factors to identify the zeros and then sketch the polynomial.

M2-18

a.  $f(x) = x^4 - 4x^3 - x^2 + 4x$

b.  $f(x) = x^4 - 10x^2 + 9$

