Warm Up

Determine each quotient.

1. $9x \div 3x$

2. $4x^2 \div 2x$

3. $x^3 \div x$

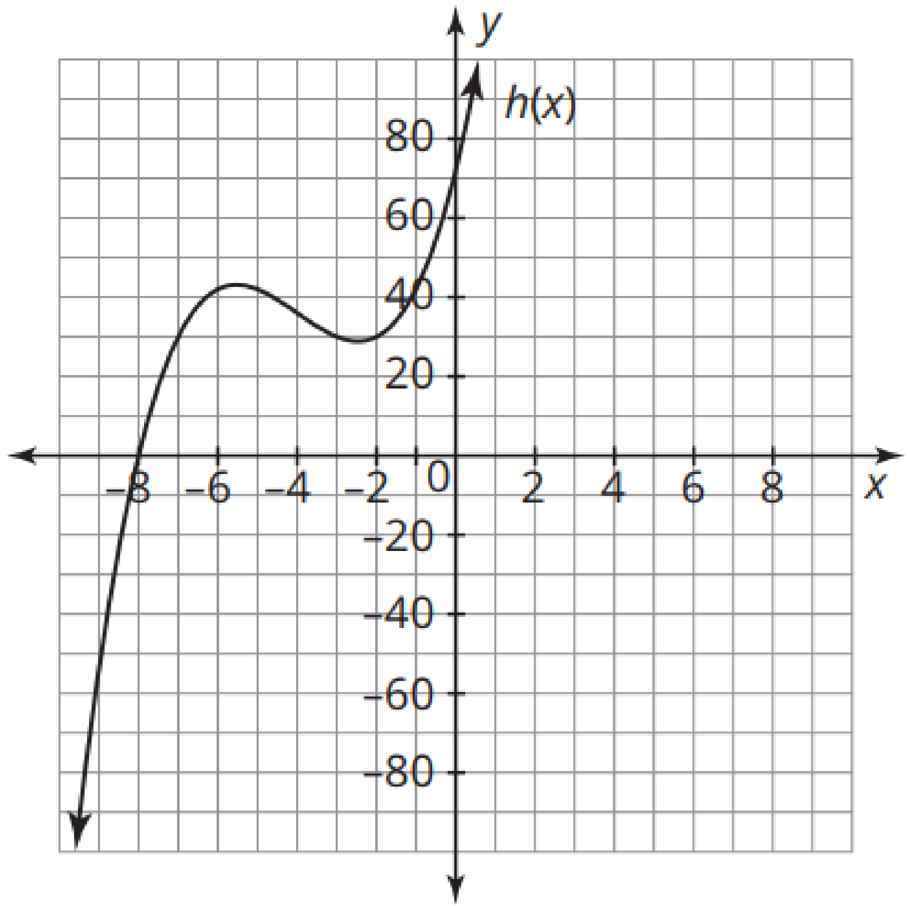
Learning Goals

- Describe similarities between polynomials and integers.
- Determine factors of a polynomial using one or more roots of the polynomial.
- Compare polynomial long division to integer long division.
- Determine factors through polynomial long division.
- Use the Remainder Theorem to evaluate polynomial equations and functions.

Key Terms

- Factor Theorem
- polynomial long division
- Remainder Theorem
- synthetic division

1. Analyze the graph of the function $h(x) = x^3 + 12x^2 + 41x + 72$.



a. Describe the key characteristics of $h(x)$.

b. Describe the number and types of zeros of $h(x)$.

The **Factor Theorem** states that a polynomial function $p(x)$ has $(x - r)$ as a factor if and only if the value of the function at r is 0, or $p(r) = 0$.

Worked Example

Consider the graph of the polynomial function $h(x) = x^3 + 12x^2 + 41x + 72$ in Question 1.

The graph appears to have a zero at $(-8, 0)$, so a possible linear factor of the polynomial is $(x + 8)$.

Determine the value of the polynomial at $x = -8$, or $h(-8)$.

$$h(-8) = (-8)^3 + 12(-8)^2 + 41(-8) + 72$$

$$= -512 + 768 + (-328) + 72$$

$$= 0$$

So, $(x + 8)$ is a linear factor of the polynomial function.

2. Consider that $d(x) = (x + 8)$, and $d(x) \cdot q(x) = h(x)$.

a. What do you know about the function $q(x)$?

b. Can you write the algebraic representation for $q(x)$? Explain your reasoning.

Worked Example

Integer Long Division	Polynomial Long Division
$3660 \div 12$ or $\frac{3660}{12}$ $\begin{array}{r} 305 \\ 12\overline{)3660} \\ -36 \\ \hline 6 \\ -0 \\ \hline 60 \\ -60 \\ \hline 0 \end{array}$	$(x^3 + 12x^2 + 41x + 72) \div (x + 8)$ or $\frac{x^3 + 12x^2 + 41x + 72}{x - 8}$ $\begin{array}{r} {}^{\text{A}}x^2 + {}^{\text{D}}4x + {}^{\text{G}}9 \\ x + 8\overline{)x^3 + 12x^2 + 41x + 72} \\ {}^{\text{B}}-(x^3 + 8x^2) \downarrow \quad \downarrow \\ \hline {}^{\text{E}}4x^2 + 41x^{\text{C}} \quad \\ -(4x^2 + 32x) \downarrow \\ \hline 9x + 72^{\text{F}} \\ {}^{\text{H}}-(9x + 72) \\ \hline \text{Remainder } 0 \end{array}$ A. Divide $\frac{x^3}{x} = x^2$. B. Multiply $x^2(x + 8)$, and then subtract. C. Bring down $41x$. D. Divide $\frac{4x^2}{x} = 4x$. E. Multiply $4x(x + 8)$, and then subtract. F. Bring down $+72$. G. Divide $\frac{9x}{x} = 9$. H. Multiply $9(x + 8)$, and then subtract.

1. Analyze the worked example that shows integer long division and polynomial long division.

   a. In what ways are the integer and polynomial long division algorithms similar?

   b. Rewrite each expression as a product of its factors.

   $3660 = \_\_\_\_\_\_ \cdot \_\_\_\_\_\_$ $\quad x^3 + 12x^2 + 41x + 72 = \_\_\_\_\_\_ \cdot \_\_\_\_\_\_$

c. Is $h(x)$ completely factored?
Explain your reasoning.

d. Rewrite the function as a product of its linear factors.

e. Determine the zeros of the function.

2. The expression $(x - 7)$ is a factor of $x^3 - 10x^2 + 11x + 70$. M2-27
Solve $0 = x^3 - 10x^2 + 11x + 70$ over the set of complex numbers.