Find the quotient of

$8x^3 - 10x^2 - x + 3$ and $x - 1$

$$
\begin{array}{r}
8x^2 - 2x - 3 \\
x - 1 \overline{\big) \; 8x^3 - 10x^2 - x + 3} \\
\underline{-8x^3 + 8x^2 \qquad\qquad} \\
-2x^2 - x \quad\;\; \\
\underline{2x^2 - 2x} \quad\;\; \\
-3x + 3 \\
\underline{3x - 3} \\
0
\end{array}
$$

Recall that you can use the difference of squares to factor a binomial of the form $a^2 - b^2$. The binomial $a^2 - b^2 = (a + b)(a - b)$.

1. Use the difference of squares to factor each binomial over the set of real numbers.

a. $x^2 - 64$

b. $x^4 - 16$

c. $x^8 - 1$

d. $x^4 - y^4$

Now let's consider expressions composed of perfect cubes, such as $f(x) = x^3 - 8$.

2. Consider Kingston's and Toby's work.

Kingston

I can use the Properties of Equality to determine the factors.

$x^3 - 8 = 0$

$x^3 = 8$

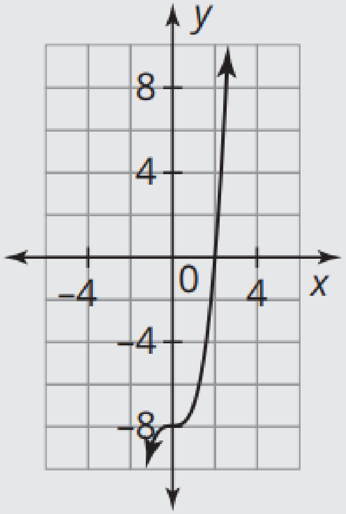
$x = 2$

$f(x) = (x - 2)(x - 2)(x - 2)$

or

$f(x) = (x - 2)^3$

Toby



I looked at the graph of $f(x) = x^3$ and could se that $x = 2$ is one of the zeros, but the other two zeros are imaginary.

a. Describe Kingston's error.

b. Use long division to factor over the set $f(x) = x^3 - 8$ of real numbers.

Worked Example

To determine the factor formula for the difference of cubes, factor out $(a - b)$ by considering $(a^3 - b^3) \div (a - b)$.

$$
\begin{array}{r}
a^2 + ab + b^2 \\
a - b \,\overline{\smash{)}\, a^3 - 0a^2b + 0ab^2 - b^3} \\
\underline{-(a^3 - a^2b)} \\
a^2b + 0ab^2 \\
\underline{-(a^2b - ab^2)} \\
ab^2 - b^3 \\
\underline{-(ab^2 - b^3)} \\
0
\end{array}
$$

Therefore, the difference of cubes can be rewritten in factored form as:

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$.

3. Use Properties of Equality to determine one zero. Then factor each polynomial function over the set of real numbers.

a. $f(x) = x^3 - 27$

b. $g(x) = x^3 + 27$

4. Determine the formula for the sum of cubes by dividing $a^3 + b^3$ by $(a + b)$.

5. Use the sum or difference of cubes to factor each binomial over the set of real numbers.

a. $x^3 + 125$

b. $8x^3 - 1$

c. $x^6 - 8$

d. $x^9 + y^9$