

Warm Up

Solve each equation for x .

$$1. \frac{10}{x} = \frac{7}{x-3}$$

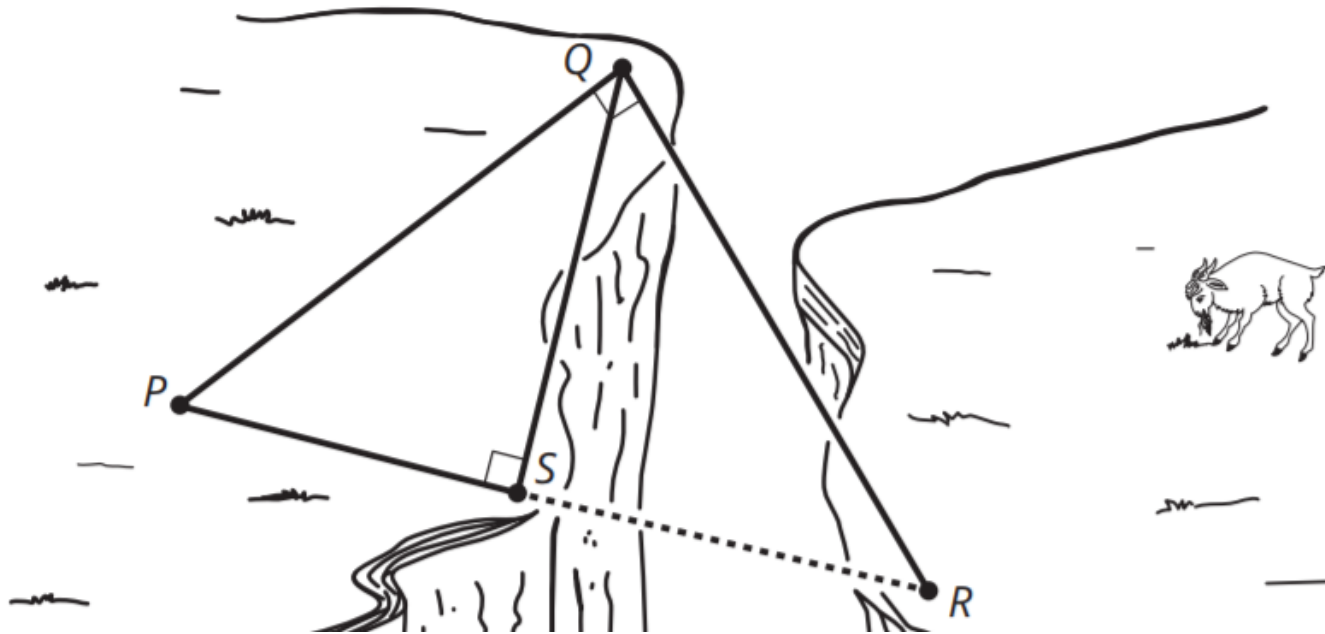
$$2. \frac{x}{2} = \frac{8}{x}$$

$$3. \frac{x}{7} = \frac{6}{x}$$

Bridging the Gap

M2-66

A bridge is needed to cross over a canyon. The dotted line segment connecting points S and R represents the bridge. The distance from point P to point S is 45 yards. The distance from point Q to point S is 130 feet. How long is the bridge?

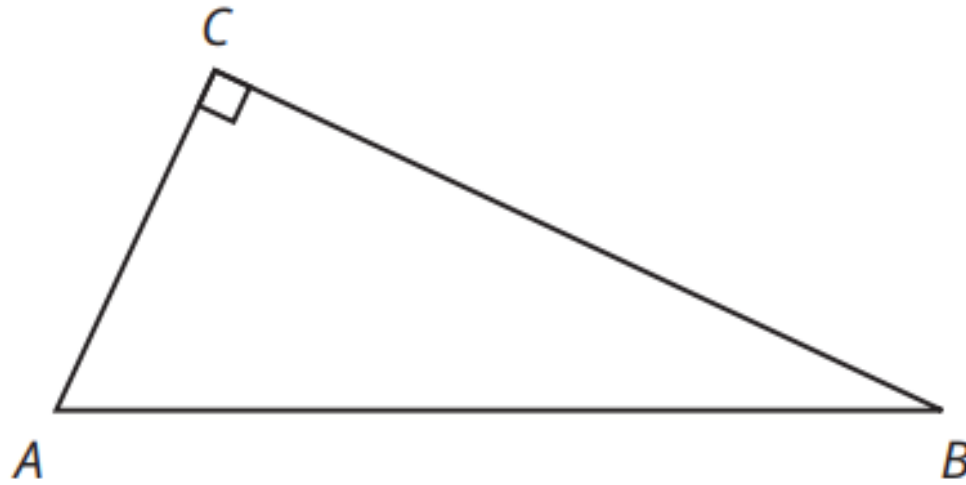


1. Construct an altitude to the hypotenuse in the right triangle ABC . Label the altitude CD .

M2-66

Remember:

An altitude is a perpendicular segment that indicates the height of a figure. It is drawn from a vertex to the opposite side or to the line that contains the opposite side.



2. Name all right triangles in the figure.
3. Trace each of the triangles on separate pieces of patty paper and label all the vertices on each triangle. Arrange the triangles so that all of the triangles have the same orientation. The hypotenuse, the shortest leg, and the longest leg should all be in corresponding positions.

4. Name each pair of triangles that are similar. Explain how you know that each pair of triangles are similar.

5. Write the corresponding sides of each pair of triangles as proportions.

The **Right Triangle/Altitude Similarity Theorem** states: "If an altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other."

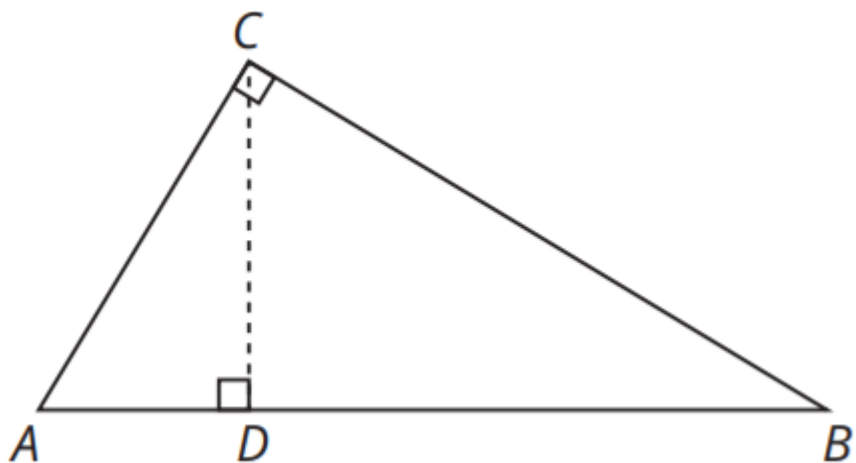
When an altitude of a right triangle is constructed from the right angle to the hypotenuse, three similar right triangles are created. This altitude is a *geometric mean*. The **geometric mean** of two positive numbers a and b is the positive number x such that $\frac{a}{x} = \frac{x}{b}$.

Two theorems are associated with the altitude to the hypotenuse of a right triangle.

The **Right Triangle Altitude/Hypotenuse Theorem** states: "The measure of the altitude drawn from the vertex of the right angle of a right triangle to its hypotenuse is the geometric mean between the measures of the two segments of the hypotenuse."

The **Right Triangle Altitude/Leg Theorem** states: "If the altitude is drawn to the hypotenuse of a right triangle, each leg of the right triangle is the geometric mean of the hypotenuse and the segment of the hypotenuse adjacent to the leg."

1. Use the diagram from the Getting Started to write a proportion to demonstrate each theorem.



Right Triangle Altitude/Hypotenuse Theorem

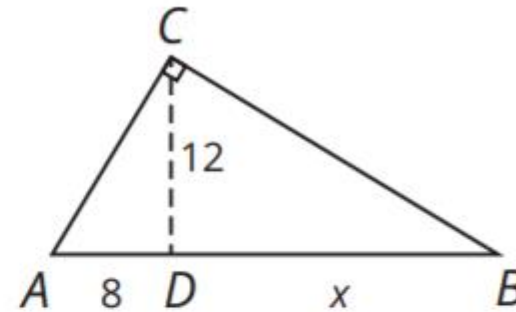
b. Right Triangle Altitude/Leg Theorem

Worked Example

M2-69

For $\triangle ABC$, the geometric mean is the altitude length of 12.

First, set up the proportion with the altitude length as the geometric mean, and then solve for x .



$$\frac{8}{12} = \frac{12}{x}$$

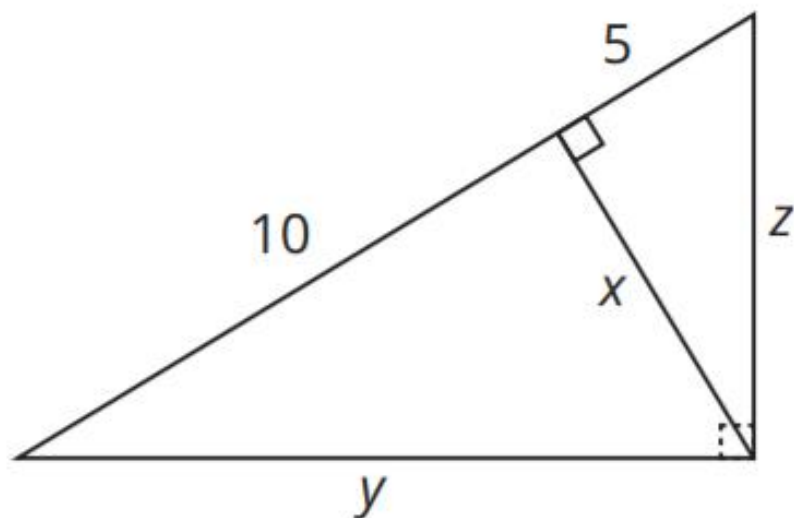
$$\frac{2}{3} = \frac{12}{x}$$

$$x = 18$$

The length of \overline{DB} is 18 units.

2. In the triangle shown, solve for x , y , and z .

M2-69



3. Solve for the length of the bridge in the Getting Started using the geometric mean.