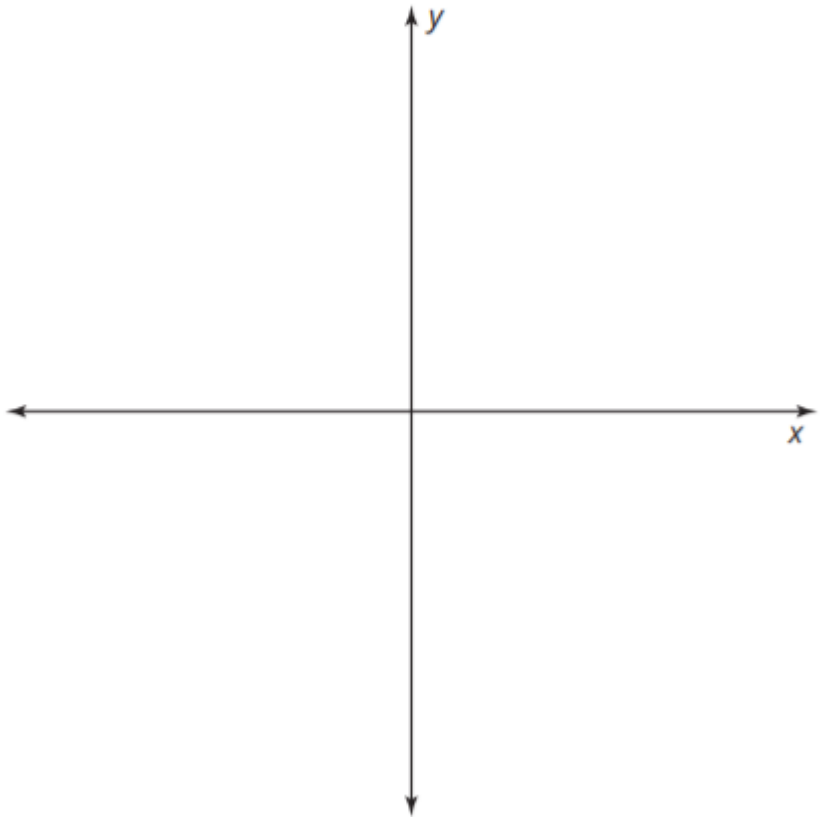
Warm Up

Consider the function $f(x) = x^4 - 13x^2 + 36$. Identify the zeros and sketch a graph of the polynomial.



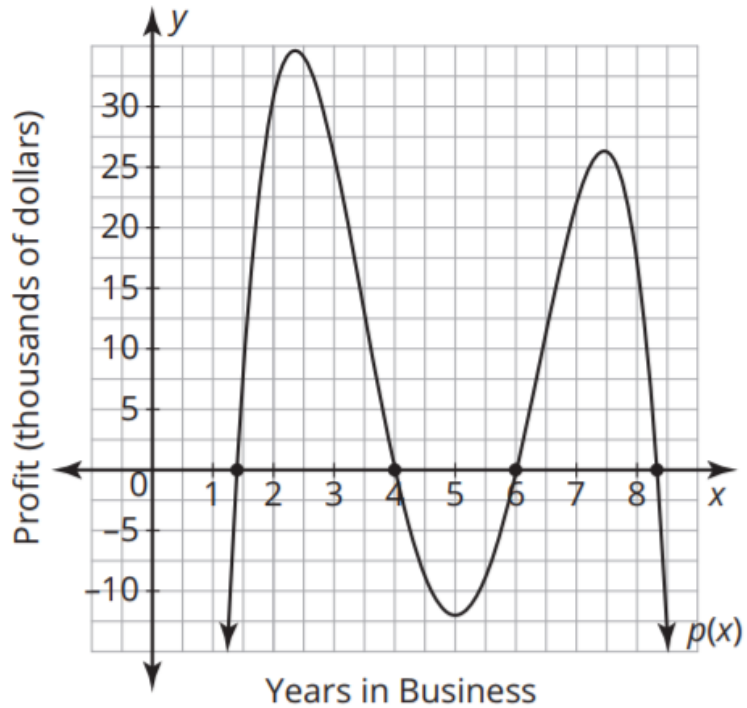
Learning Goals

- Represent problem situations using polynomial inequalities.
- Determine solutions to polynomial inequalities algebraically and graphically.

Lawn Enforcement is a small landscaping company. It has a profit model that can be represented by the function

$$p(x) = -x^4 + 19.75x^3 - 133.25x^2 + 351.25x - 280.75$$

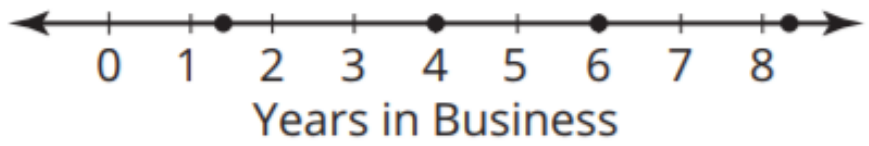
where profit, in thousands of dollars, is a function of time, in years, the company has been in business. Let's analyze $p(x)$ represented on a graph.



1. **What do the zeros of the function represent?**

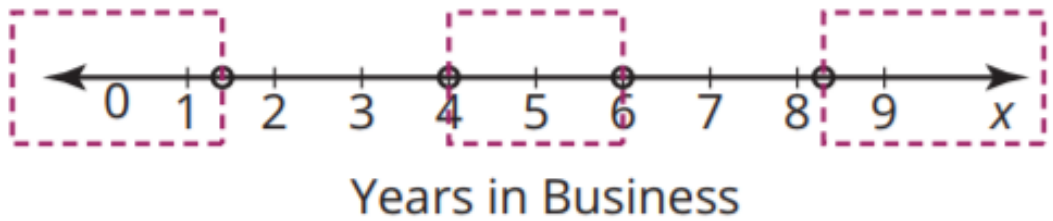
2. **Approximate the intervals of increase and decrease. What do these intervals represent?**

Each point on the number line represents the years in business when Lawn Enforcement's profit was 0.



The function $p(x) = 0$ when $x = 1.4, 4, 6, 8.3$.

The regions on the number line enclosed in dashed boxes represent the years in business when Lawn Enforcement's profit was less than 0.



The function $p(x) < 0$ when $\begin{cases} x < 1.4 \\ 4 < x < 6 \\ x > 8.3 \end{cases}$.

1. **Analyze the worked example.**

   a. **Why were the points changed to open circles on the number line to represent the years in business when $p(x) < 0$.**

   b. **Circle the parts of the graph on the coordinate plane that represent where $p(x) > 0$. Then circle the intervals on the number line that represent the years in business where $p(x) > 0$. Finally identify the set of x-values to complete the sentence and explain your answer in terms of this problem situation.**

      The function $p(x) > 0$ when _____.

c. Draw a solid box around the segment(s) where $p(x) > 35{,}000$. Then identify the set of x-values to complete the sentence. Finally, explain your answer in terms of this problem situation.

The function $p(x) > 35{,}000$ when _____.

Solving polynomial inequalities is very similar to solving linear inequalities.

1. Samson, Kaley, Paco, and Sal each solved the quadratic inequality $2x^2 + 14x < -24$.

Samson

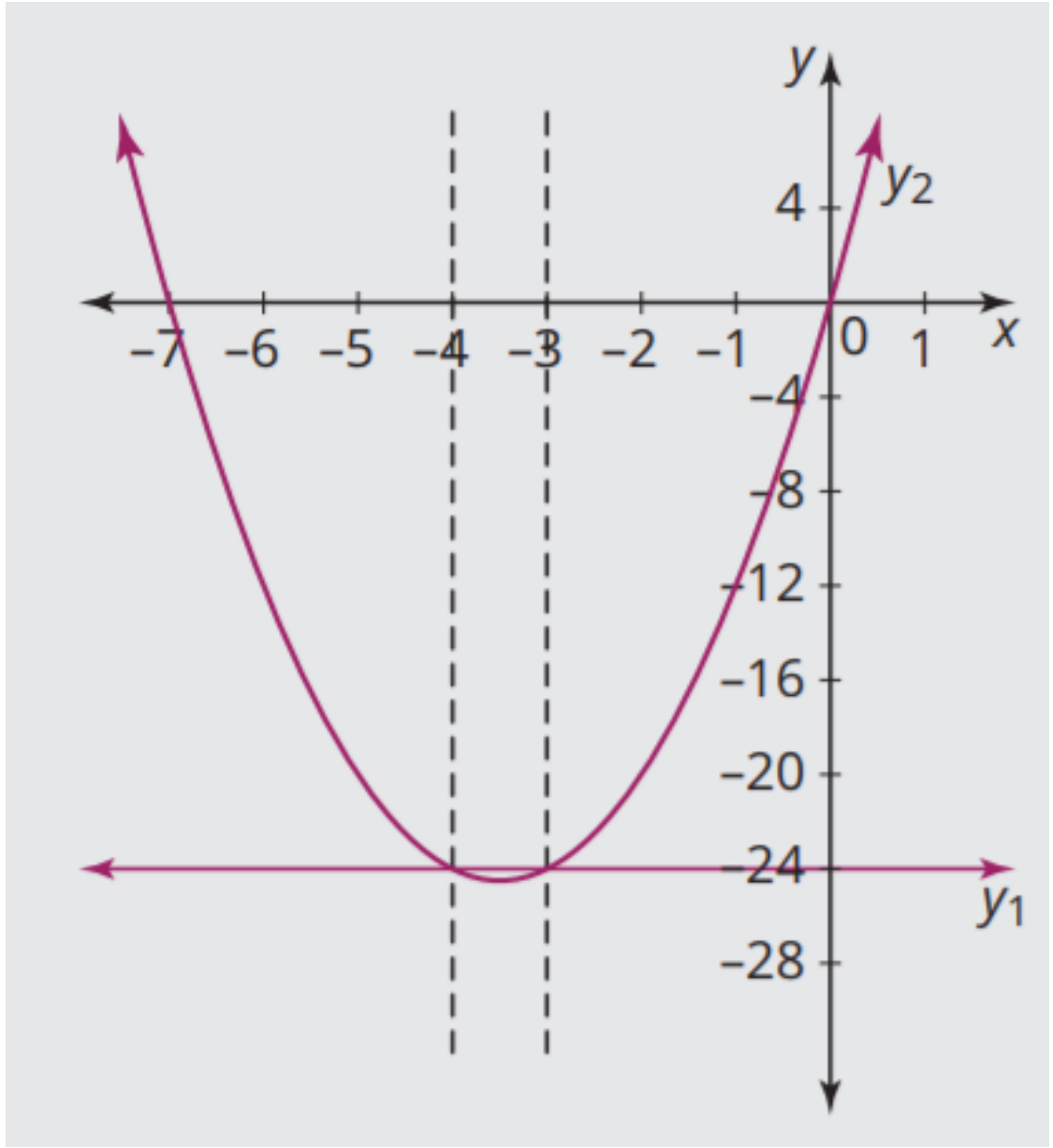
I graphed both sides of the inequality.

$$y_1 = -24$$
$$y_2 = 2x^2 + 14x$$

I drew vertical dashed lines at the two points where the graphs intersect. I can then determine from the graph that the x-values of $2x^2 + 14x$ that generate values less than -24 are between -4 and -3. Therefore the solution to the inequality is $-4 < x < -3$.

The symbols $>$ or $<$ are represented with dotted lines, and $\geq$ or $\leq$ are represented with solid lines.

When you are determining which region(s) to shade, look at y-values above or below the boundary line, depending on the inequality sign.


y
x
y2
y1
4
0
1
−4
−8
−12
−16
−20
−24
−28
−7
−6
−5
−4
−3
−2
−1

Paco

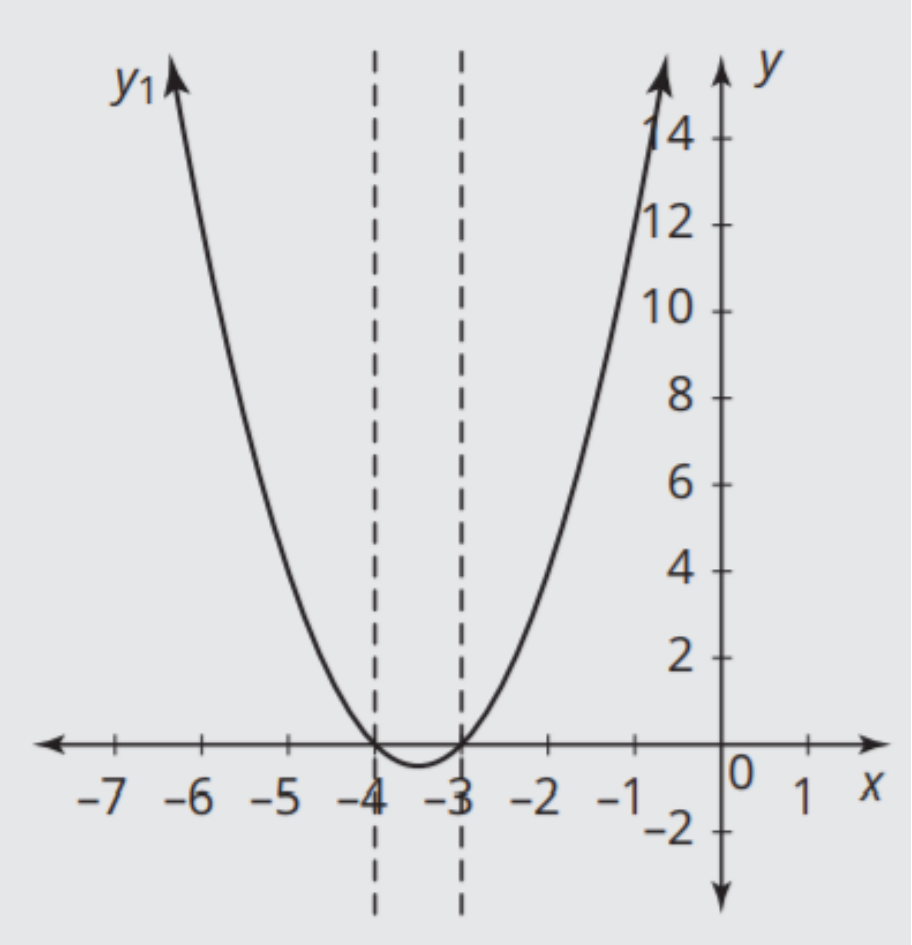
I added 24 to both sides of the inequality because I wanted one side to be equal to 0. Then, I graphed that inequality.

$$y_1 = 2x^2 + 14x + 24$$

I drew vertical dashed lines where the graph crosses the x-axis.

I can then determine from the graph that the x-values of $2x^2 + 14x + 24$ that generate values less than 0 are between -4 and -3.

Therefore the solution to the inequality is $-4 < x < -3$.


y_1
y
14
12
10
8
6
4
2
−7
−6
−5
−4
−3
−2
−1
0
1
x
−2

2. Solve $18 \leq 3x^2 + x$. Show your work algebraically and graphically.