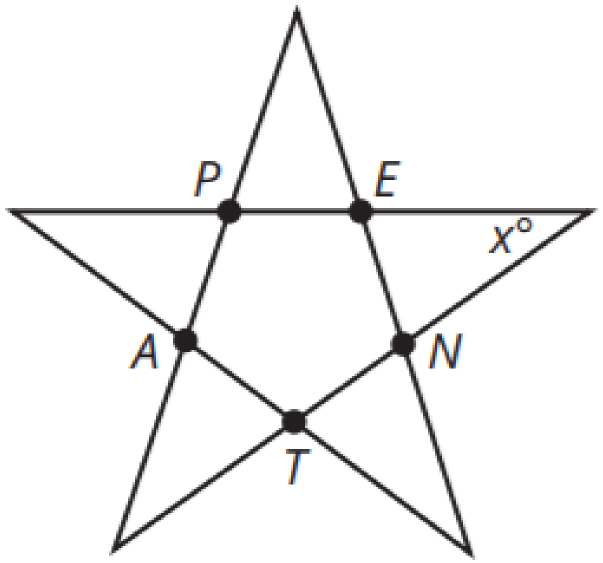
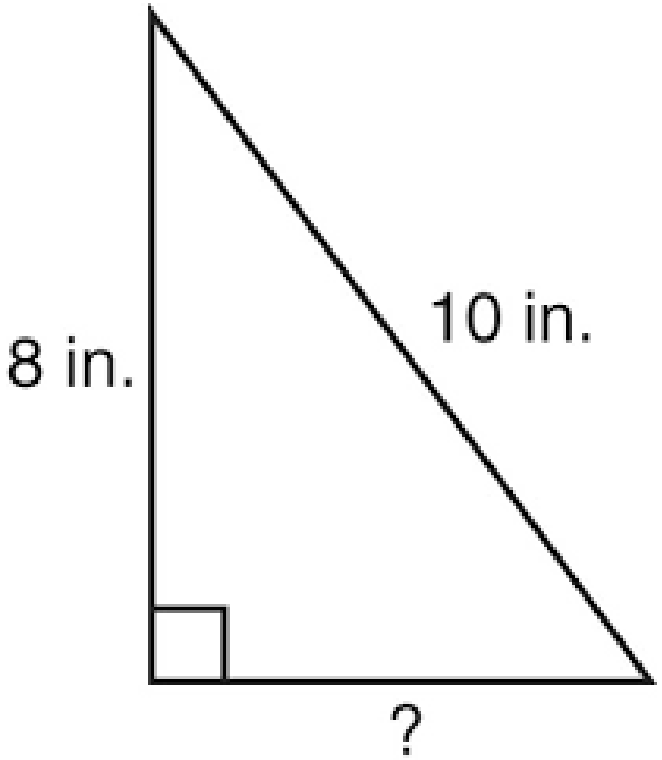
Warm-up

1) *PENTA* is a regular pentagon. Solve for *x*.



2)



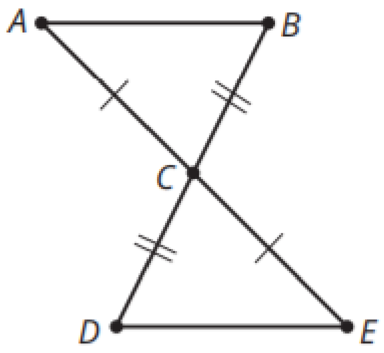
Mappings Matter

You have proven theorems related to triangle congruence. You know that two triangles are congruent when:

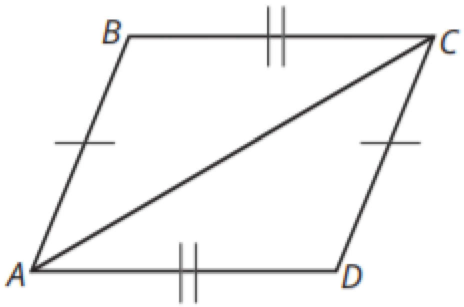
- Three corresponding sides are congruent (SSS).
- Two corresponding sides and the included angle are congruent (SAS).
- Two corresponding angles and the included side are congruent (ASA).

1. Use Side-Angle-Side (SAS), Side-Side-Side (SSS), or Angle-Side-Angle (ASA) to explain why the triangles in each pair are congruent. Explain your reasoning.

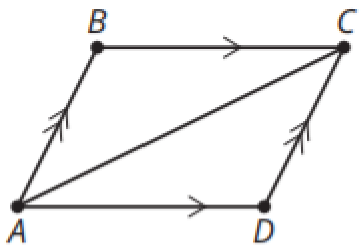
a.



b.



c.



ACTIVITY 4.1 The Perpendicular Bisector Theorem

You know that if two triangles are congruent, then each part of one triangle is congruent to the corresponding part of the other triangle. **Corresponding parts of congruent triangles are congruent**, abbreviated **CPCTC**, is often used as a reason in proofs. CPCTC states that corresponding angles or sides in two congruent triangles are congruent. This reason can be used only after you have proved that the triangles are congruent.

1. Consider $\triangle CPS$ and $\triangle WPD$ in the figure shown. Suppose $\overline{CW}$ and $\overline{SD}$ bisect each other.

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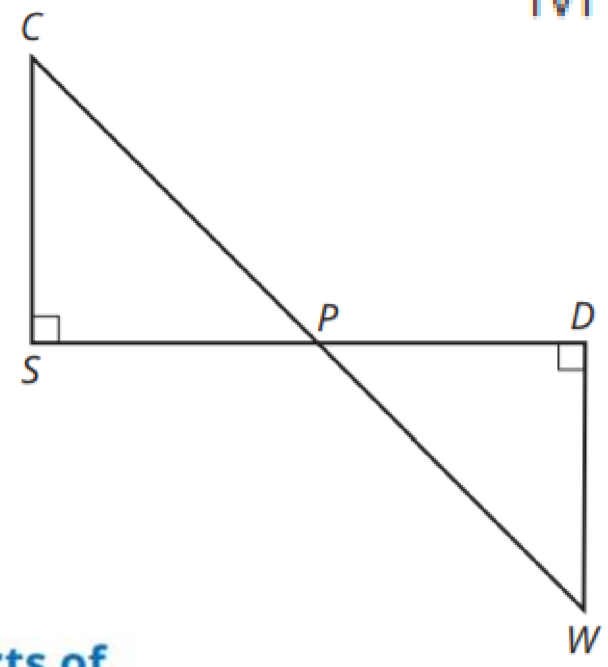


a. Explain how you can demonstrate that the triangles are congruent.

b. Use CPCTC to identify the congruent corresponding parts of the two triangles.

Draw and Deduce
How can you mark the diagram to reflect the given information?

2. **Consider $\overline{AB}$.**

a. **Construct the perpendicular bisector of $\overline{AB}$. Then, draw a line segment from any one point on the bisector to each endpoint of the line segment, *A* and *B*.**

b. **Use your construction to write a proof plan to prove the Perpendicular Bisector Theorem. Then complete the proof.**

Because you have proved this relationship is true, you can now refer to it as a theorem. The **Perpendicular Bisector Theorem** states: “Points on a perpendicular bisector of a line segment are equidistant from the segment’s endpoints.”

3. Consider the converse of the Perpendicular Bisector Theorem.

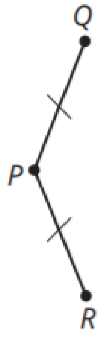
   a. State the Perpendicular Bisector Theorem as a conditional statement using if-then.

   b. State the converse of Perpendicular Bisector Theorem.

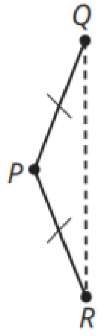
Analyze the proof of the converse of the Perpendicular Bisector Theorem.

Worked Example

Given: Points Q and R are equidistant from point P.



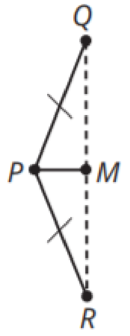
An auxiliary line segment, $\overline{QR}$, can be drawn to form an isosceles triangle, $\triangle RPQ$.



Construct the midpoint of $\overline{QR}$, point M. Another auxiliary line segment, $\overline{PM}$, can be drawn, connecting the midpoint with point P.

The two triangles, $\triangle PQM$ and $\triangle PRM$ are congruent triangles by SSS Congruence.

This means that $\angle PMQ$ and $\angle PMR$ are congruent by CPCTC. And since these two angles are congruent and form a linear pair, they are both 90° angles.



Thus, point P lies on the perpendicular bisector of $\overline{QR}$.

4. Consider the worked example. How would you know that point P lies on the perpendicular bisector of $\overline{QR}$? Explain your reasoning.

Because this relationship has been proved true, you can now refer to it as a theorem. The **Perpendicular Bisector Converse Theorem** states: If a point is equidistant from the endpoints of a line segment, then the point lies on the perpendicular bisector of the segment." This theorem can be useful when proving theorems about right triangles.

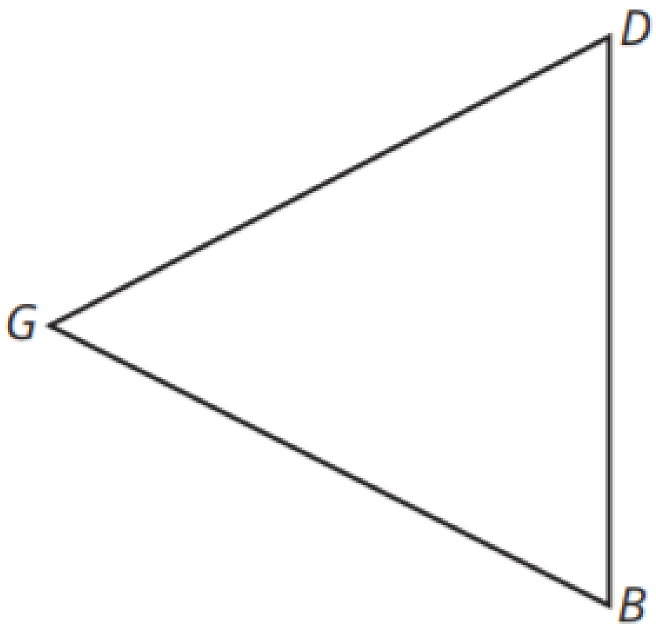
CPCTC makes it possible to prove other theorems.

You have explored the relationship between the sides and angles of an isosceles triangle. Specifically, you conjectured that if the two sides of a triangle are congruent, then the angles opposite these sides are congruent. The Isosceles Triangle Base Angles Theorem states: “If two sides of a triangle are congruent, then the angles opposite these sides are congruent.””

1. **Consider the diagram to prove your conjecture using a paragraph proof.**

   Given: $\overline{GB} \cong \overline{GD}$

   Prove: $\angle B \cong \angle D$



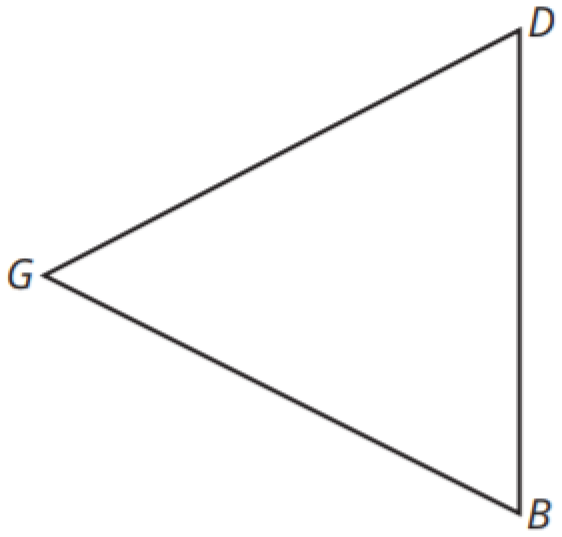
Because you have proved this relationship is true, you can now refer to it as a theorem. The **Isosceles Triangle Base Angles Theorem** states: “If two sides of a triangle are congruent, then the angles opposite these sides are congruent.”

2. **Consider the converse of the Isosceles Triangle Base Angles Theorem.**

   a. **State the converse as a conjecture.**

   b. **Consider the diagram to prove your conjecture using a paragraph proof.**

   Given: $\angle B \cong \angle D$
   Prove: $\overline{GB} \cong \overline{GD}$



Because you have proved this relationship is true, you can now refer to it as a theorem. The **Isosceles Triangle Base Angles Converse Theorem** states: “If two angles of a triangle are congruent, then the sides opposite these angles are congruent.”