Warm Up

Evaluate each expression for $x = 2$ and $x = -3$.

1. x^4

2. $-x^2 + 1$

3. $(-x)^2 - 1$

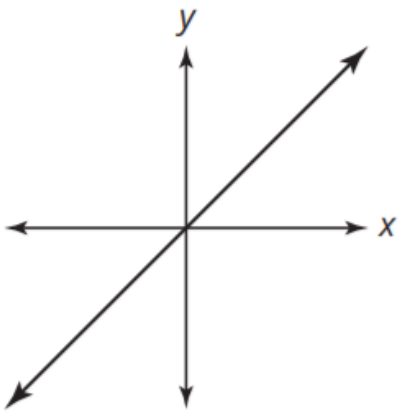
4. x^8

Flat in the Middle

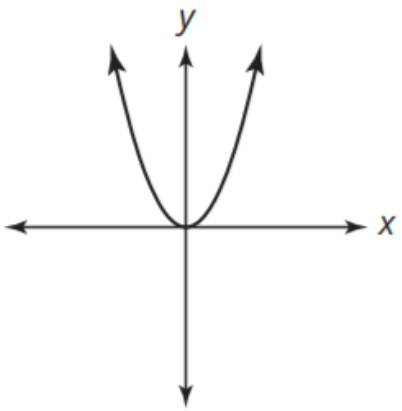
You have studied linear functions, quadratic functions, and now you will explore more polynomial functions. A common type of polynomial function, which you have also studied, is a *power function*. A **power function** is a function of the form $P(x) = ax^n$, where n is a non-negative integer.

1. **Consider each power function and its graph in the sequence shown.**

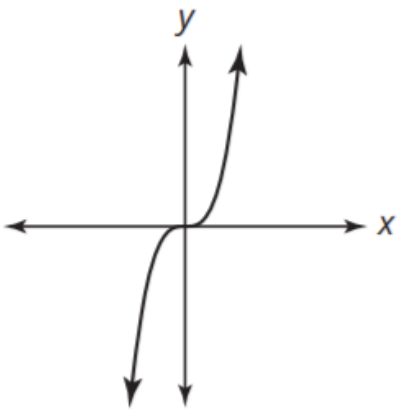
$y = x$



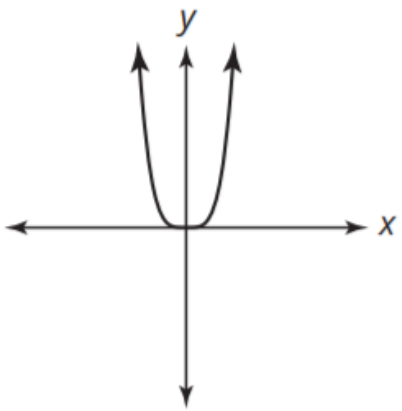
$y = x^2$



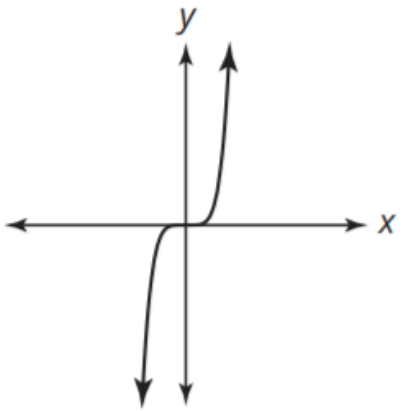
$y = x^3$



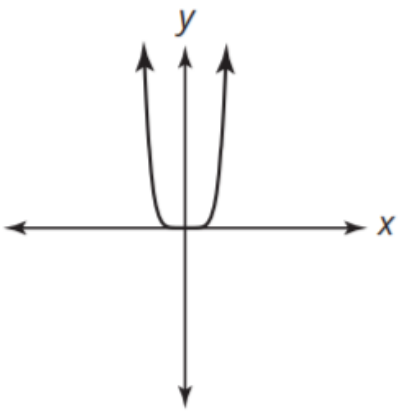
$y = x^4$



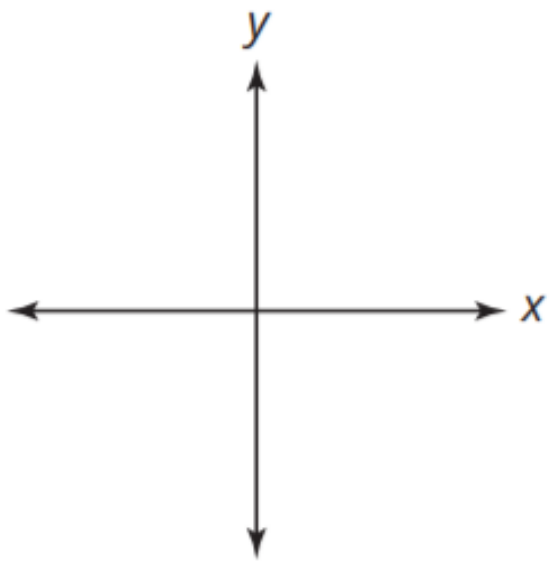
$y = x^5$

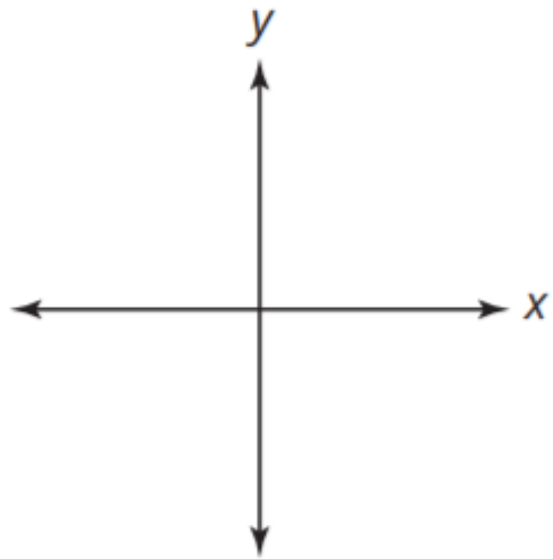


$y = x^6$



a. Sketch and label the next two graphs in the sequence.





b. State any observations or patterns that you notice about the graphs in the sequence.

Consider the power functions graphed in the Getting Started. The tables show values of each function for different domain values.

Odd Degree			
x	x^1	x^3	x^5
-2	-2	-8	-32
-1	-1	-1	-1
$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{8}$	$-\frac{1}{32}$
0	0	0	0
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{8}$	$\frac{1}{32}$
1	1	1	1
2	2	8	32

Even Degree			
x	x^2	x^4	x^6
-2	4	16	64
-1	1	1	1
$-\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{16}$	$\frac{1}{64}$
0	0	0	0
$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{16}$	$\frac{1}{64}$
1	1	1	1
2	4	16	64

1. For both the odd- and even-degree functions, observe the change in the values between -1 and 1. How does this change compare with the change in the values less than -1 and greater than 1?

2. Observe the behavior of each function on either side of 0, using the tables and graphs.

   a. Describe the behavior of a function of odd degree on either side of 0.

   b. Describe the behavior of a function of even degree on either side of 0.

3. For both the odd- and even-degree functions:

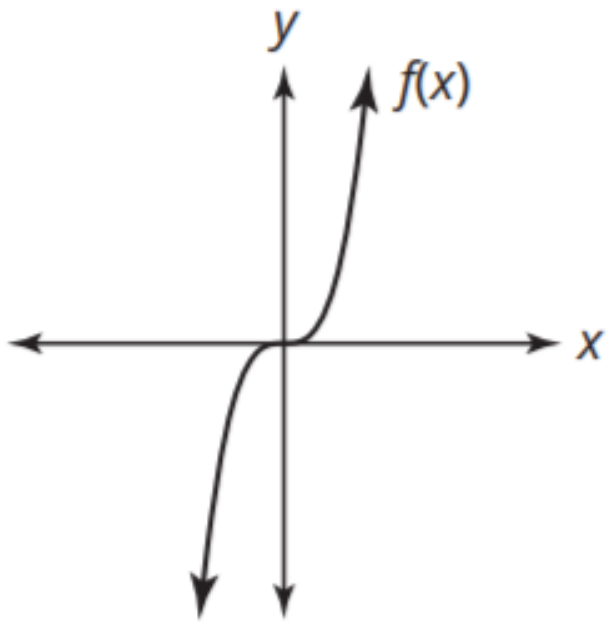
   a. Explain why the graphs flatten as the degree increases, for values of x between -1 and 1.

   b. Explain why the graphs steepen as the degree increases, for values of x less than -1 and greater than 1.

The **end behavior** of a graph of a function is the behavior of the graph as x approaches infinity and as x approaches negative infinity.

Worked Example

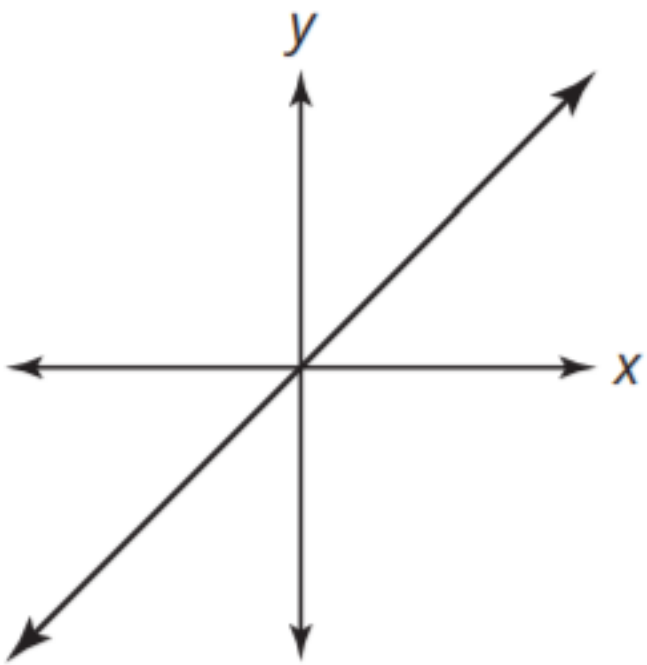
You can write the end behavior of the polynomial function shown using this notation.



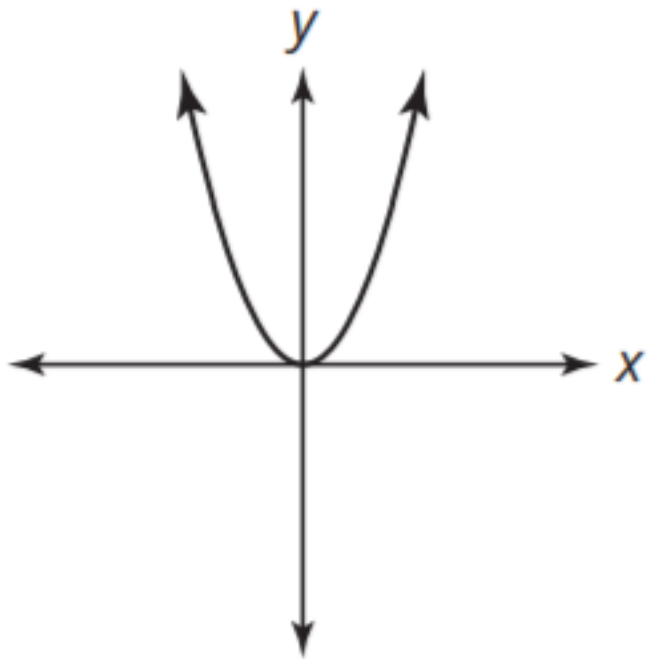
As $x \to \infty$, $f(x) \to \infty$.
As $x \to -\infty$, $f(x) \to -\infty$.

1. Consider the sequence of graphs shown.

$f_1(x) = x$



$f_2(x) = x^2$



$f_3(x) = x^3$