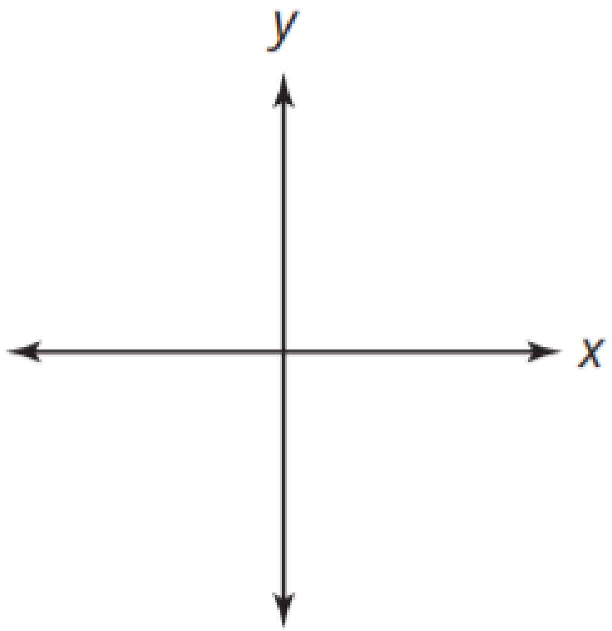
1) Graph $f\left(x\right)=x^{3}$

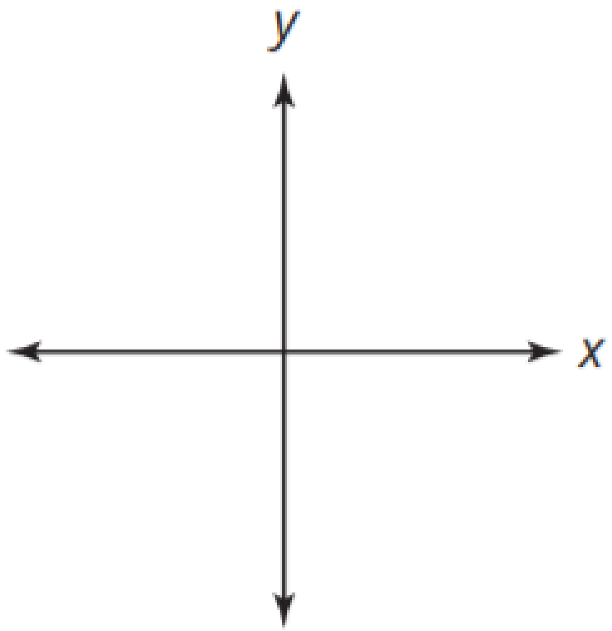
Odd Degree			
x	x^1	x^3	x^5
-2	-2	-8	-32
-1	-1	-1	-1
$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{8}$	$-\frac{1}{32}$
0	0	0	0
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{8}$	$\frac{1}{32}$
1	1	1	1
2	2	8	32

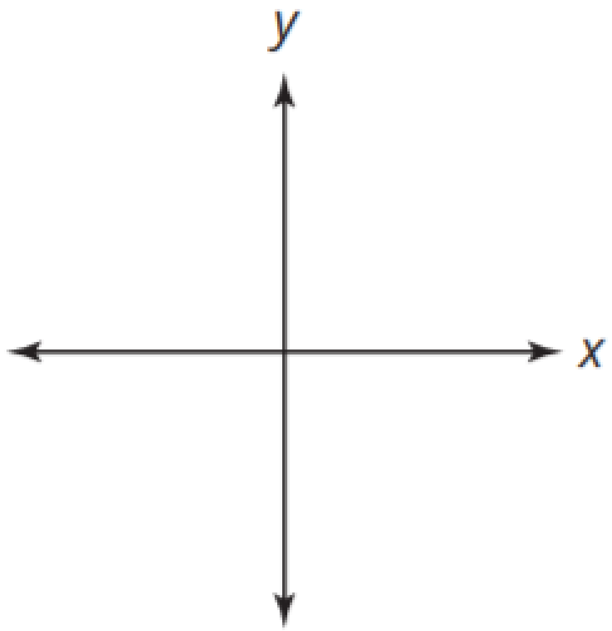
Even Degree			
x	x^2	x^4	x^6
-2	4	16	64
-1	1	1	1
$-\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{16}$	$\frac{1}{64}$
0	0	0	0
$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{16}$	$\frac{1}{64}$
1	1	1	1
2	4	16	64

b. **What happens when the *B*-value is negative? Write each function in terms of *x*, and then sketch it.**

$f_1(-x) =$ ____________ $f_2(-x) =$ ____________ $f_3(-x) =$ ____________







c. **Complete the table to describe the end behavior for any polynomial function.**

	Odd Degree Power Function	Even Degree Power Function
$A > 0$		
$A < 0$		
$B > 0$		
$B < 0$		

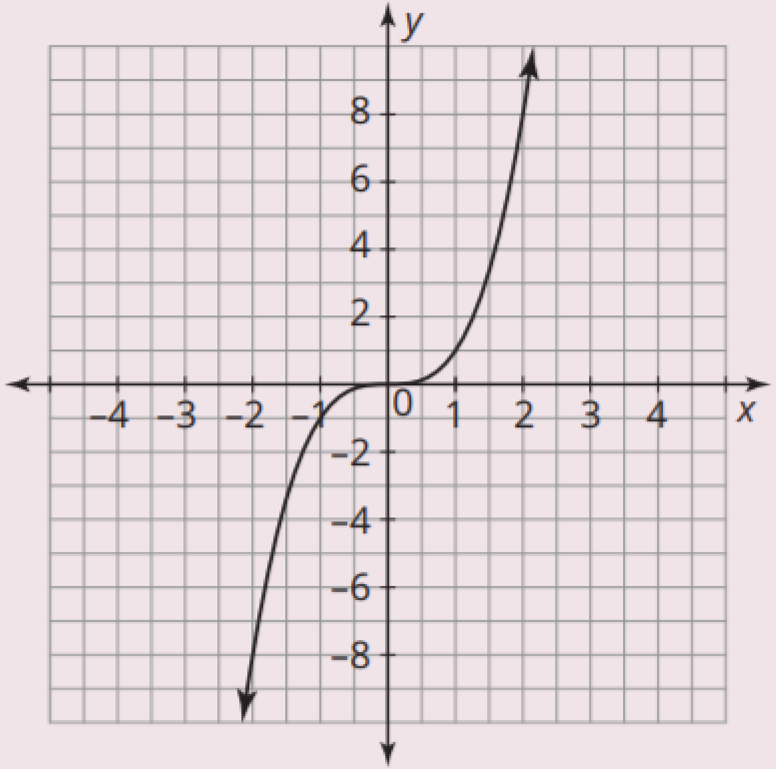
ACTIVITY 1.3 Investigating Characteristics of Even and Odd Functions

If a graph is **symmetric about a line**, the line divides the graph into two identical parts. Special attention is given to the line of symmetry when it is the *y*-axis as it tells you that the function is even.

Think about:

The graph of $y = x^2$ is symmetric about the line $x = 0$.

1. **Analyze the graph shown.**



Olivia says that the graph has no line of symmetry because if she reflected the graph across the x- or y-axis, it would not be a mirror image.

Randall says that the graph has no line of symmetry because if he looks at the x-values 1 and -1, the y-values are not the same, so there can't be symmetry about the y-axis. Also, if he looks at the y-values 8 and -8, the x-values are not the same, so there can't be symmetry about the x-axis.

Shedrick said that there is some type of symmetry. He looks at the point $(2, 8)$ and notices that the point $(-2, -8)$ is also on the graph. Likewise he looks at the point $(1, 1)$ and notices that the point $(-1, -1)$ is also on the graph. He concluded that it must have a reflection across the x- and y-axis at the same time.

Who's correct? Explain your reasoning.

The graph of an odd degree basic power function is *symmetric about a point*, in particular the origin. A function is **symmetric about a point** if each point on the graph has a point the same distance from the central point, but in the opposite direction. Special attention is given when the central point is the origin as it determines that the function is odd. When the point of symmetry is the origin, the graph is reflected across the x-axis and the y-axis. If you replace both (x, y) with $(-x, -y)$, the function remains the same.

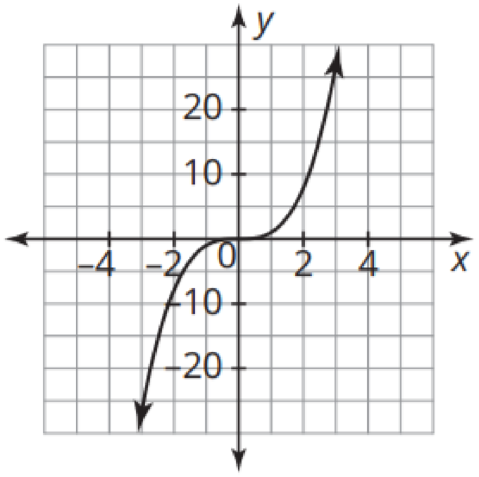
Worked Example

Consider the function $f(x) = x^3$. You can think of the point of symmetry about the origin as a double reflection.

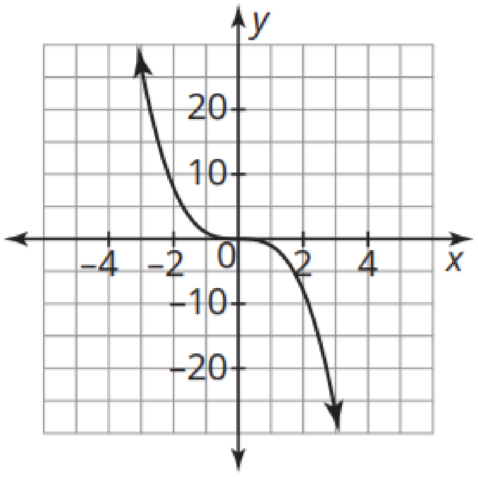
$$f_1(x) = x^3$$

$$\begin{aligned} f_2(x) &= f_1(-x) \\ &= (-x)^3 \end{aligned}$$

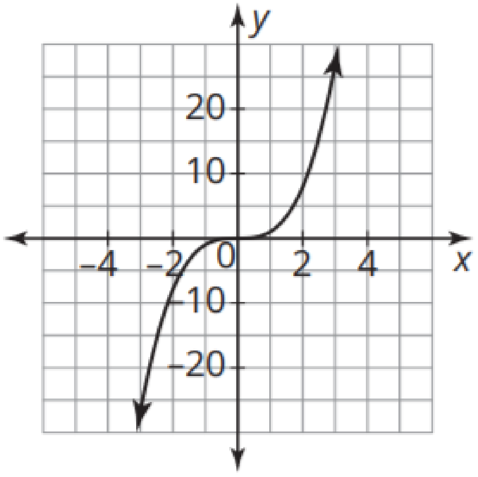
$$\begin{aligned} f_3(x) &= -f_2(x) \\ &= -((-x)^3) \\ &= x^3 \end{aligned}$$



The function $f_1(x)$ is shown.



The function $f_1(x)$ is reflected across the y-axis to produce f_2.



The function $f_2(x)$ is reflected across the x-axis to produce f_3.

An **even function** has a graph symmetric about the y-axis, thus $f(x) = f(-x)$.

An **odd function** has a graph symmetric about the origin, thus $f(x) = -f(-x)$.

2. Use what you know about *A* and *B*-value transformations to describe even and odd functions in your own words.

Odd and even functions are NOT the same as odd- and even-degree functions. A function must have an odd or even degree. But a function is not necessarily odd or even.

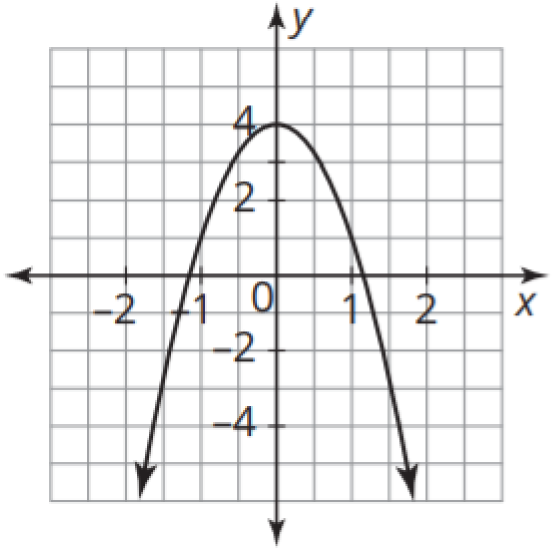
3. Explain why Claire is correct. Use function notation to write her conclusion.

> Claire
> The reflection of an odd function across the y-axis produces the same graph as its reflection across the x-axis.

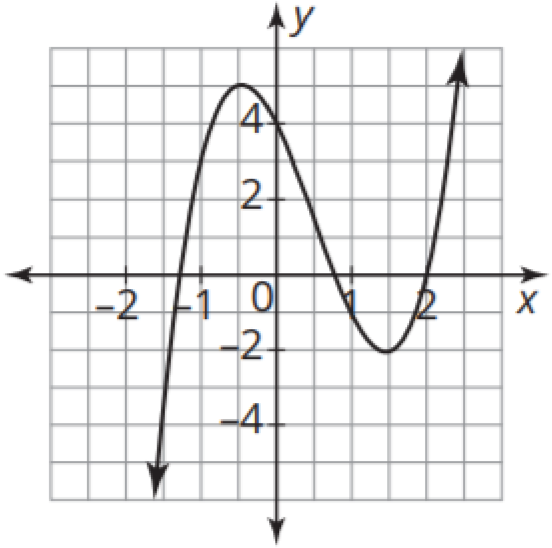
You determine algebraically whether a function is even by evaluating the function at $f(-x)$. If $f(-x) = f(x)$, then the function is even. You can also evaluate the function at $-f(x)$. If $-f(-x) = f(x)$, or $-f(x) = f(-x)$, then the function is odd.

4. State whether the graph of each function shown is even, odd, or neither.

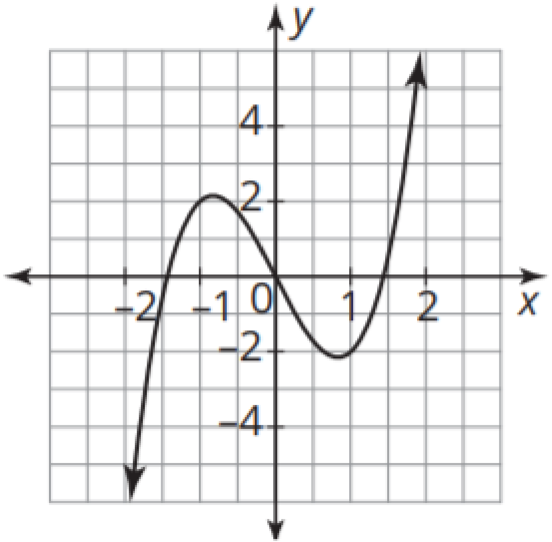
a.



b.



c.



5. Determine algebraically whether each function is even, odd, or neither. Describe the end behavior of the function.

a. $f(x) = 2x^3 - 3x$

b. $g(x) = 6x^2 + 10$

c. $h(x) = x^3 - 3x^2 - 2x + 7$