## EXAMPLE 1 Finding the Domain of a Rational Function

Find the domain of $f$ and use limits to describe its behavior at values) of $x$ not in its domain.

$$
f(x)=\frac{1}{x-2}
$$


$D:(-\infty, 2) \cup(2, \infty)$

| $X$ | $Y_{1}$ |  |
| :---: | :--- | :---: |
| 2 | ERROR |  |
| 1.99 | -1001 |  |
| 1.98 | -50 |  |
| 1.97 | -33.33 |  |
| 1.96 | -25 |  |
| 1.95 | -20 |  |
| 1.94 | -16.67 |  |
| $Y_{1}=1 /[X-2]$ |  |  |


$R:(-\infty, 0) \cup(0, \infty) \lim _{x \rightarrow 2^{-}} f(x)=\infty$ and $\lim _{x \rightarrow 2^{+}} f(x)=\infty$.

EXAMPLE 2 Transforming the Reciprocal Function
Describe how the graph of the given function can be obtained by transforming the graph of the reciprocal function $f(x)=1 / x$. Identify the horizontal and vertical asymptotes and use limits to describe the corresponding behavior. Sketch the graph of the function.
(a) $g(x)=\frac{(2)}{x+3}$
(b) $h(x)=\frac{3 x-7}{x-2}$
(2) $\left(\frac{1}{x+3}\right)=-f(x+3)$

VS by factor of 2
HL 3 units

$$
\begin{aligned}
& \text { (3) }-\frac{1}{x-2}=-f(x-2)+(3 .)^{K} V U T \\
& \text { reflection } \\
& \text { over }
\end{aligned}
$$

EXAMPLE 3 Finding Asymptotes
Find the horizontal and vertical asymptotes of $f(x)=\left(x^{2}+2\right)\left(x^{2}+1\right)$. Use limits to describe the corresponding behavior of $f$.


$$
1+\frac{1}{x^{2}+1}
$$

no


$$
\begin{aligned}
\lim _{x \rightarrow \infty} f(x)=\lim _{x \rightarrow-\infty} f(x) & =1, \\
\text { AA @ } y & =1
\end{aligned}
$$

and behavior

## Graph of a Rational Function

The graph of $y=f(x) / g(x)=\left(a_{n},(\sqrt{1})+\cdots\right) /\left(b_{m}:(m)+\cdots\right)$ has the following characteristics:

1. End behavior asymptote:
 If $n<m$, the end behavior asymptote is the horizontal asymptote $y=0$. ff $n=m$. the end behavior asymptote is the horizontal asymptote $y=a_{n} / b_{m}$. f $n>m$ ) the end behavior asymptote is the quotient polynomial function $y=q(x)$, where $f(x)=g(x) q(x)+r(x)$. There is no horizontal asymptote.
2. Vertical asymptotes: These occur at the zeros of the denominator provided that the zeros are not also zeros of the numerator of equal or greater multiplicity.
3. $x$-intercepts: These occur at the zeros of the numerator, which are not also zeros of the denominator.
4. $\boldsymbol{y}$-intercept: This is the value of $f(0)$, if defined.

## EXAMPLE 4 Graphing a Rational Function

Find the asymptotes and intercepts of the function $f(x)=x^{3} /\left(x^{2}-9\right)$ and graph the


If the end behavior asymptote of a rational function is a slant line, we call it a slant asymptote


## EXAMPLE 4 Graphing a Rational Function

Find the asymptotes and intercepts of the function $f(x)=x^{3} /\left(x^{2}-9\right)$ and graph the function.

$$
f(x)=\frac{x^{3}}{x^{2}-9}=x+\frac{9 x}{x^{2}-9}
$$

$$
\frac{x^{3}}{x-9}
$$



If the end behavior asymptote of a rational function is a slant line, we call it a slant asymptote


## EXAMPLE 5 Analyzing the Graph of a Rational Function

Find the intercepts, asymptotes, use limits to describe the behavior at the vertical asymptotes, and analyze and draw the graph of the rational function

$\lim _{x \rightarrow-2^{-}} f(x)=-\infty, \lim _{x \rightarrow-2^{+}} f(x)=\infty, \lim _{x \rightarrow 3^{-}} f(x)=-\infty$, and $\lim _{x \rightarrow 3^{+}} f(x)=\infty$.

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(a) $g(x)=\frac{2}{x+3}$
(b) $h(x)=\frac{3 x-7}{x-2}$
$2\left(\frac{1}{x+3}\right)=2 f(x+3)$

$$
\begin{array}{r}
3 \\
x - 2 \longdiv { 3 x - 7 } \\
\frac{3 x-6}{-1}
\end{array}
$$

$$
3-\frac{1}{x-2}=-f(x-2)+3
$$

