Solving Rational Equations

When we multiply or divide an equation by an expression containing variables, the resulting equation may have solutions that are *not* solutions of the original equation. These are **extraneous solutions**. For this reason we must check each solution of the resulting equation in the original equation.

$\frac{1}{2} + \frac{1}{3}$ 6

EXAMPLE 1 Solving by Clearing Fractions

Solve $\frac{x}{1} + \frac{3}{x} = \frac{4}{1}$.

LCD $\Rightarrow x \cdot 1 = x$

multiply both sides by LCD

$$x \cdot x + \cancel{x} \cdot \frac{3}{\cancel{x}} = x \cdot 4$$

$$x^2 + 3 = 4x$$

$$x^2 - 4x + 3 = 0$$

$$(x-1)(x-3) = 0$$

1, 3

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EXAMPLE 1 Solving by Clearing Fractions

Solve $x + \frac{3}{x} = 4$.

$\boxed{1, 3}$

$1 + \frac{3}{1} = 4$ ✓

$3 + \frac{3}{3} = 4$ ✓

EXAMPLE 2 Solving a Rational Equation

Solve $x + \dfrac{1}{x-4} = 0.$

LCD $= x - 4$

$(x-4) \cdot x + \cancel{(x-4)} \cdot \dfrac{1}{\cancel{(x-4)}} = (x-4)\,0$

$2+\sqrt{3}$

$2-\sqrt{3}$

$x(x-4)+1=0$

$x^2-4x+1=0$

$\dfrac{4 \pm \sqrt{16-4}}{2} = \dfrac{4 \pm \sqrt{12}}{2}$

$= \dfrac{4 \pm 2\sqrt{3}}{2} = \boxed{2 \pm \sqrt{3}}$

EXAMPLE 3 Eliminating Extraneous Solutions

Solve the equation $\dfrac{2x}{x-1} + \dfrac{1}{x-3} = \dfrac{2}{x^2-4x+3}$.

$(x-3)(x-1)$

$LCD = (x-1)(x-3)$

$$\cancel{(x-1)}(x-3)\cdot\frac{2x}{\cancel{(x-1)}} + (x-1)\cancel{(x-3)}\cdot\frac{1}{\cancel{(x-3)}} = \cancel{(x-1)(x-3)}\cdot\frac{2}{\cancel{x^2-4x+3}}$$

$$2x^2 - 6x + x - 1 = 2$$

$$2x^2 - 5x - 3 = 0$$

$$(2x+1)(x-3) = 0$$

$$\boxed{x = -\frac{1}{2}},\ \cancel{3}$$

EXAMPLE 4 Eliminating Extraneous Solutions

Solve

$$\frac{x-3}{x}+\frac{3}{x+2}+\frac{6}{x^2+2x}=0.$$

$LCD = x(x+2)$

$(x+2)(x-3)+3x+6=0$

$x^2-x\cancel{-6}+3x\cancel{+6}=0$

$x^2+2x=0$

$x(x+2)=0$

$\cancel{0}, \cancel{-2}$

NO solution