Solving Rational Equations

When we multiply or divide an equation by an expression containing variables, the resulting equation may have solutions that are *not* solutions of the original equation. These are **extraneous solutions**. For this reason we must check each solution of the resulting equation in the original equation.

EXAMPLE 1 Solving by Clearing Fractions

Solve $\frac{x}{1} + \frac{3}{x} = \frac{4}{1}$.

LCD $\Rightarrow x \cdot 1 = x$

multiply both sides by LCD, x

$x \cdot x + \cancel{x} \cdot \frac{3}{\cancel{x}} = x \cdot 4$

$x^2 + 3 = 4x$

set $= 0$

$x^2 - 4x + 3 = 0$

$(x-1)(x-3) = 0$

$1, 3$

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EXAMPLE 1 Solving by Clearing Fractions

Solve $x + \frac{3}{x} = 4$.

$1 + \frac{3}{1} = 4$ ✓

$3 + \frac{3}{3} = 4$ ✓

$\boxed{1, 3}$

EXAMPLE 2 Solving a Rational Equation

Solve $x + \frac{1}{x-4} = 0$.

LCD $\Rightarrow$ $x-4$

$$(x-4)\cdot x + \cancel{(x-4)}\cdot\frac{1}{\cancel{x-4}} = (x-4)\cdot 0$$

$$x^2 - 4x + 1 = 0$$

$$\frac{4 \pm \sqrt{16-4}}{2} = \frac{4 \pm \sqrt{12}}{2}$$

$$\frac{4}{2} \pm \frac{2\sqrt{3}}{2} \qquad \frac{4 \pm 2\sqrt{3}}{2} = 2 \pm \sqrt{3}$$

EXAMPLE 3 Eliminating Extraneous Solutions

Solve the equation $\dfrac{2x}{x-1} + \dfrac{1}{x-3} = \dfrac{2}{x^2 - 4x + 3}$.

$LCD = (x-3)(x-1)$

$(x-3)(x-1)$

$$(x-3)\cancel{(x-1)} \cdot \frac{2x}{\cancel{(x-1)}} + \cancel{(x-3)}(x-1) \cdot \frac{1}{\cancel{(x-3)}} = \cancel{(x-3)(x-1)} \cdot \frac{2}{\cancel{(x-3)(x-1)}}$$

$$2x(x-3) + (x-1) = 2$$

$$2x^2 - 6x + x - 1 = 2$$

$$2x^2 - 5x - 3 = 0$$

$$(2x+1)(x-3) = 0$$

$$x = \frac{-1}{2}, 3$$

EXAMPLE 3 Eliminating Extraneous Solutions

Solve the equation $\dfrac{2x}{x-1} + \dfrac{1}{x-3} = \dfrac{2}{x^2 - 4x + 3}$.

$\cancel{3}$

$\dfrac{2(3)}{3-1} + \dfrac{1}{\cancel{3-3}}$

$-\dfrac{1}{2}$

0.38

$\dfrac{2(-\frac{1}{2})}{-\frac{1}{2}-1} + \dfrac{1}{-\frac{1}{2}-3} = \dfrac{2}{(-\frac{1}{2})^2 - 4(-\frac{1}{2}) + 3}$

$\dfrac{\frac{1}{3}}{\frac{2}{2}}$

$\dfrac{-1}{3/2} + \dfrac{1}{-\frac{7}{2}} = \dfrac{2}{\frac{1}{4} + 2 + 3}$

$\dfrac{2}{3} - \dfrac{2}{7} = \dfrac{2}{5.25}$

EXAMPLE 4 Eliminating Extraneous Solutions

Solve

LCD $\Rightarrow x(x+2)$

$$\frac{x-3}{x}+\frac{3}{x+2}+\frac{6}{x^2+2x}=0.$$

$x(x+2)$

$$(x+2)(x-3)+3x+6=0$$

$$x^2-x-\cancel{6}+3x+\cancel{6}=0$$

$$x^2+2x=0$$

$$x(x+2)=0$$

$$\cancel{0} \quad \cancel{-2}$$

both extraneous

NO solution!