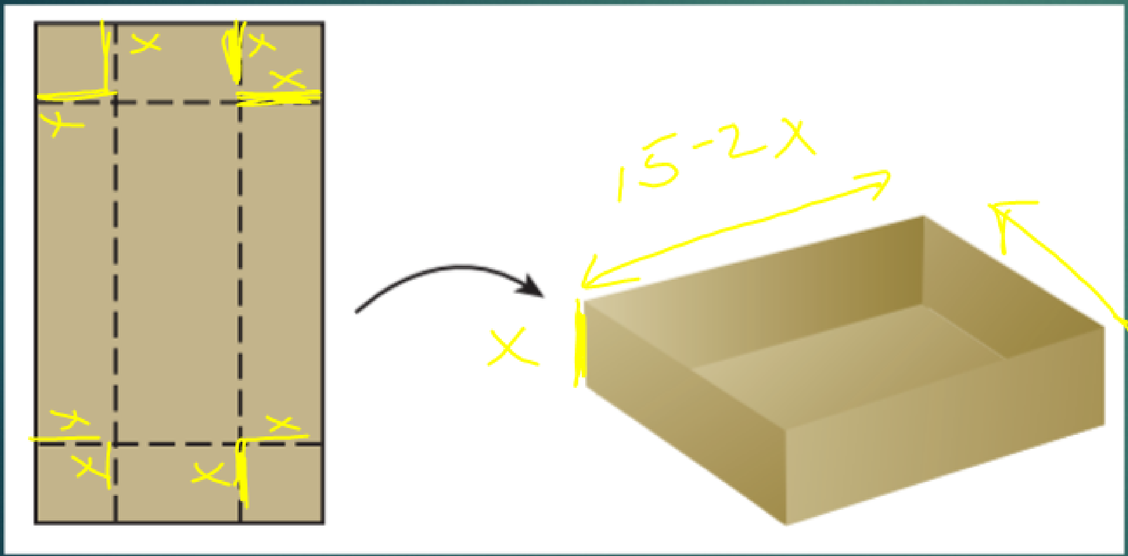
A Maximum Value Problem

A square of side x inches is cut out of each corner of an 8 in. by 15 in. piece of card-board and the sides are folded up to form an open-topped box

(a) Write the volume V of the box as a function of x.

$V = l \cdot w \cdot h$



$V(x) = (15-2x)(8-2x)x$

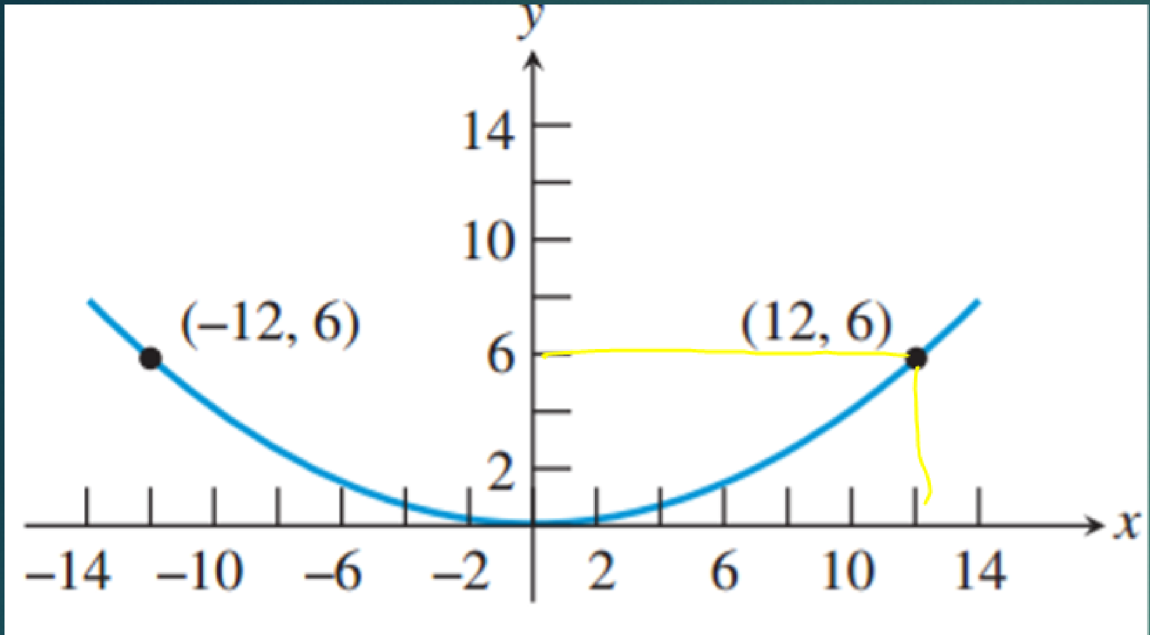
(b) Find the domain of V as a function of x.

implied domain $(0,4)$

(c) How big should the cut-out squares be in order to produce the box of maximum volume?

if $x = 1\frac{2}{3}$ in

maximum volume is $90.7\ in^3$



$$y = x^2$$

$$y = a x^2$$

$$6 = a(12)^2$$

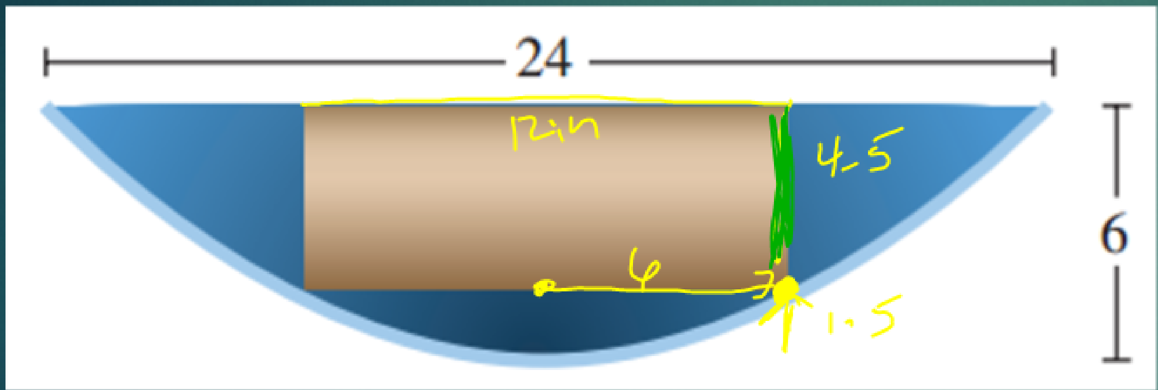
$$\frac{6}{144} = \frac{144a}{144}$$

$$a = \frac{1}{24}$$

$$f(x) = \frac{1}{24} x^2$$

Protecting an Antenna

A small satellite dish is packaged with a cardboard cylinder for protection. The parabolic dish is 24 in. in diameter and 6 in. deep, and the diameter of the cardboard cylinder is 12 in. How tall must the cylinder be to fit in the middle of the dish and be flush with the top of the dish?



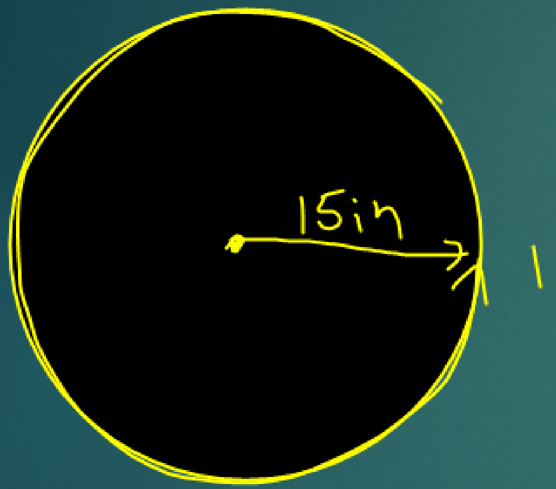
$$f(x) = \frac{1}{24}x^2$$

$$f(6) = \frac{1}{24}(6)^2 = \frac{36}{24} = 1.5 \text{ in}$$

$$6 - 1.5 = 4.5$$

Letting Units Work for You

How many rotations does a 15 in. (radius) tire make per second on a sport utility vehicle traveling 70 mph?



$$\frac{70 \text{ miles}}{1 \text{ hour}} \times \frac{1 \text{ hour}}{60 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ sec}} \times \frac{5280 \text{ ft}}{1 \text{ mile}} \times \frac{12 \text{ in}}{1 \text{ ft}}$$

$$\frac{1232 \text{ in}}{1 \text{ sec}} \times \frac{1 \text{ rot.}}{30\pi \text{ in}} = 129 \frac{\text{rot.}}{\text{sec}}$$

$$C = 2\pi r$$
$$C = 30\pi$$