# Warm Up

Use the Distributive Property to rewrite each expression.

1. $a(2a - 1)(5 + a)$

2. $(9 - x)(x + 3)$

3. $b^2(10 - b) + b^2$

4. $(w - 2)(w + 3)(w + 1)$

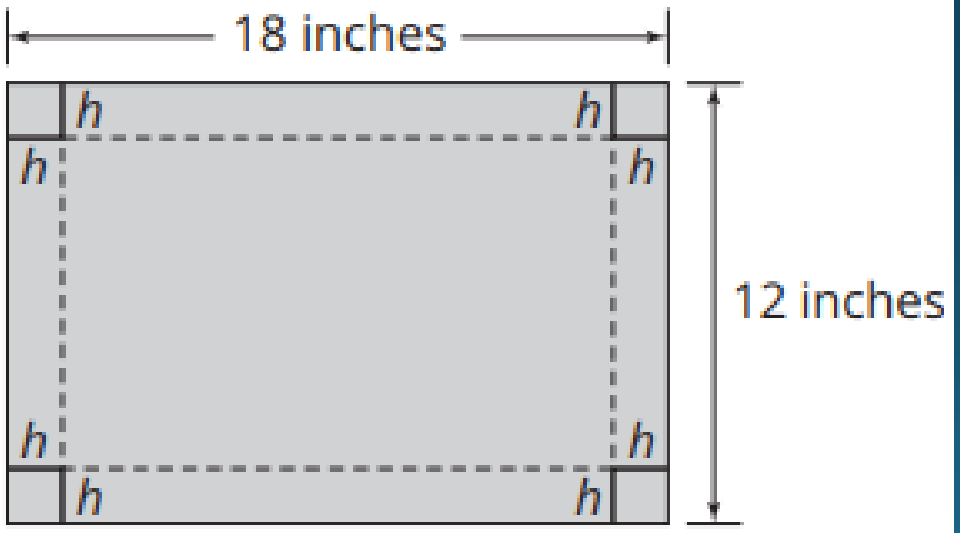
## Key Terms

- cubic function
- relative maximum
- relative minimum

# Our Business Is Growing

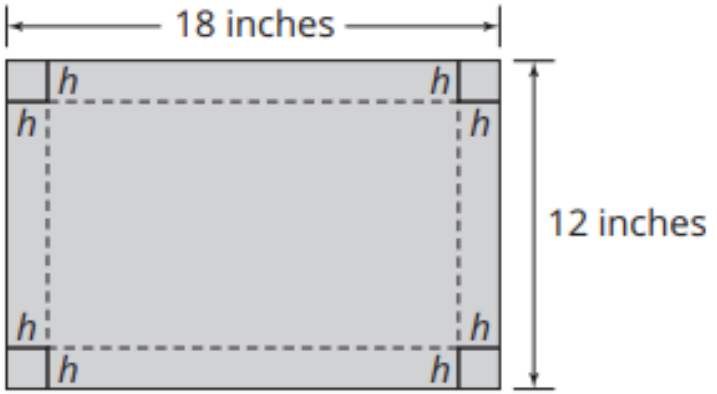
The Plant-A-Seed Planter Company produces planter boxes. To make the boxes, a square is cut from each corner of a rectangular copper sheet. The sides are bent to form a rectangular prism without a top. Cutting different sized squares from the corners results in differently sized planter boxes. Plant-A-Seed takes sales orders from customers who request a sized planter box.

Each rectangular copper sheet is 12 inches by 18 inches. In the diagram, the solid lines indicate where the square corners are cut, and the dotted lines represent where the sides are bent for each planter box.



1. ***Complete the table given each planter box is made from a 12 in by 18 inch copper sheet. Include an expression for each planter box's height, width, length, and volume for a square corner side of length h.***

| Square Corner Side Length (inches) | Height (inches) | Width (inches) | Length (inches) | Volume (cubic inches) |
|---|---|---|---|---|
| 0 | | | | |
| 1 | | | | |
| 2 | | | | |
| 3 | | | | |
| 4 | | | | |
| 5 | | | | |
| 6 | | | | |
| 7 | | | | |
| *h* | | | | |



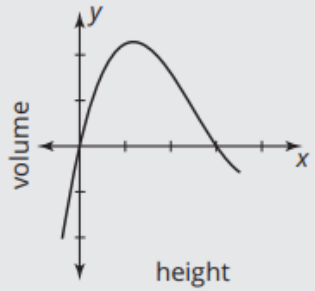
2. **Analyze the relationship between the height, length, and width of each planter box. What are the dimensions of the largest sized square corner that can be cut to make a planter box? Explain your reasoning.**


3. **Write a function $V(h)$ to represent the volume of the planter box in terms of the corner side length $h$.**

A **cubic function** is a function that can be written in the general form $f(x) = ax^3 + bx^2 + cx + d$, where $a \neq 0$.

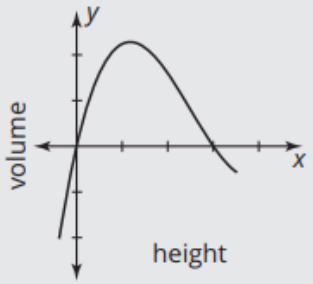
**1. Louis, Ahmed, and Heidi each used graphing technology to analyze the volume function, $V(h)$, and to sketch the graph. The disagree about the shape of the graph.**

Louis



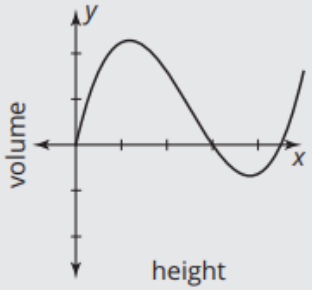

The graph increases and then decreases. It is a parabola.

Ahmed




The graph lacks a line of symmetry, so it can't be a parabola.

Heidi




I noticed the graph curves back up so it can't be a parabola.

2. Represent the cubic function using graphing technology with the settings $[-10, 15] \times [-400, 400]$.

   a. Describe the key characteristics of the graph.

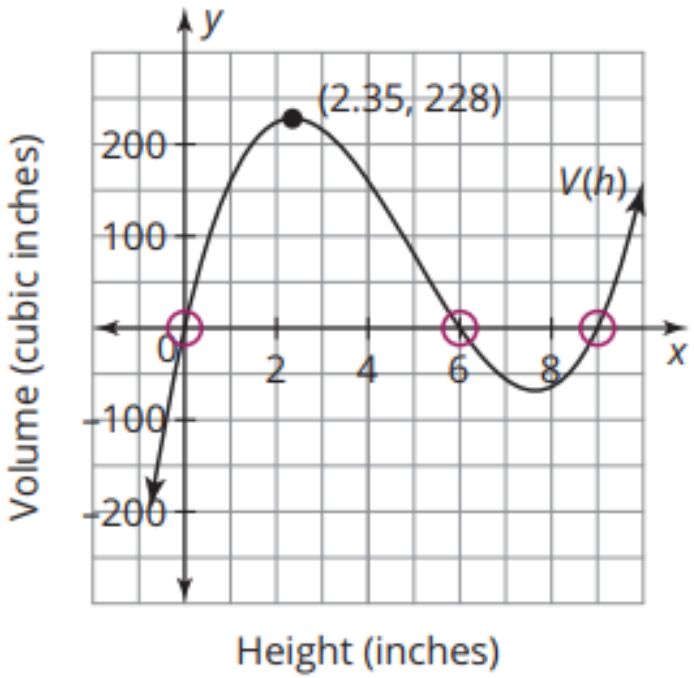
   b. What is the maximum volume of a planter box? State the dimensions of this planter box. Explain your reasoning.

   c. Identify the domain of the function $V(h)$. Is the domain the same or different in terms of the context of this problem? Explain your reasoning.

d. Identify the range of the function $V(h)$.
Is the range the same or different in terms of the context of this problem? Explain your reasoning.

e. What do the $x$-intercepts represent in this problem situation?
Do these values make sense in terms of this problem situation?
Explain your reasoning.

A graph may reveal different key characteristics within a given domain. The function $V(h) = h(12 - 2h)(18 - 2h)$ has $x$-intercepts at $x = 0$, $x = 6$, and $x = 9$.




As the input values for height increase, the output values for volume approach infinity. Therefore, the function doesn't have a maximum; however, the point (2.35, 228) is a *relative maximum* within the domain interval of (0, 6). A **relative maximum** is the highest point in a particular section of a graph.

Similarly, as the values for height decrease, the output values approach negative infinity. Therefore, a *relative minimum* occurs at (7.65, −68.16). A **relative minimum** is the lowest point in a particular section of a graph.

The function $V(h)$ represents all of the possible volumes for a given height $h$. A horizontal line is a powerful tool for working backwards to determine the possible values for the height when the volume is known.

Suppose a customer ordered a particular planter box with a volume of 100 cubic inches, but did not specify the height of the planter box.