I. Key Characteristics of Rational Functions

A. Determine whether each function is a rational function or not a rational function. If the function is not rational, explain why.

1. $f(x) = x^2 - 6x + 2$

2. $h(x) = \dfrac{x}{x-3}$
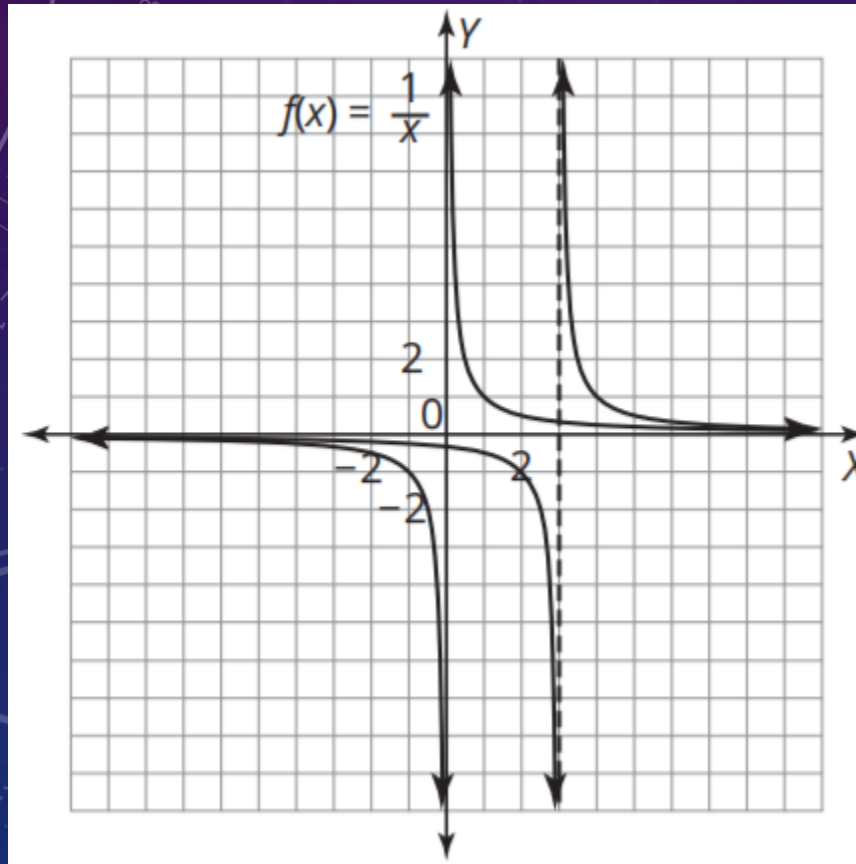
3. $r(x) = \left(\dfrac{1}{3}\right)^x$

II. Transformations of Rational Functions

A. The function $f(x) = \frac{1}{x}$ is shown on each coordinate plane. Determine whether the other function shown is the graph of $g(x)$, $p(x)$, or $q(x)$. Explain your reasoning.

1. $g(x) = \dfrac{1}{x - 3}$

$p(x) = \dfrac{1}{x + 3}$

$q(x) = \dfrac{1}{x} + 3$

$$f(x) = \frac{1}{x}$$

Transformations of Rational Functions

A. The function $f(x) = \frac{1}{x}$ is shown on each coordinate plane. Determine whether the other function shown is the graph of $g(x)$, $p(x)$, or $q(x)$. Explain your reasoning.

2. $g(x) = \frac{1}{x-3}$

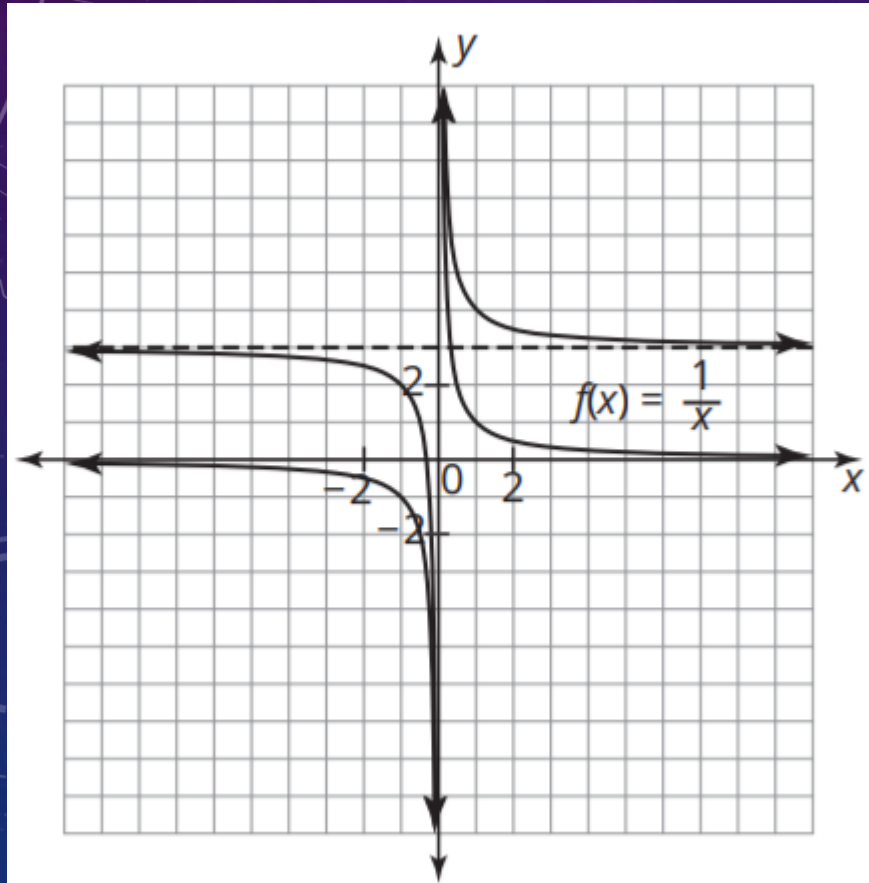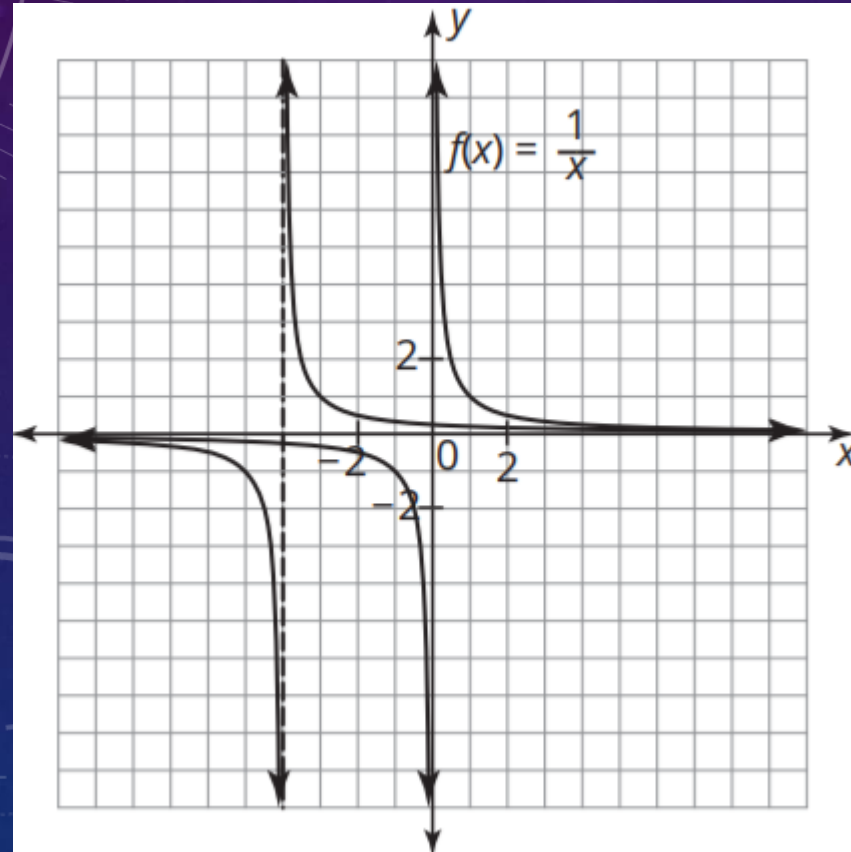$p(x) = \frac{1}{x+3}$

$q(x) = \frac{1}{x} + 3$

Transformations of Rational Functions

A. The function $f(x) = \frac{1}{x}$ is shown on each coordinate plane. Determine whether the other function shown is the graph of $g(x)$, $p(x)$, or $q(x)$. Explain your reasoning.

3. $g(x) = \dfrac{1}{x - 4}$

$p(x) = \dfrac{1}{x + 4}$

$q(x) = \dfrac{1}{x} - 4$

III. Graphical Discontinuities

B. Determine whether the graph of each rational function has a vertical asymptote, a removable discontinuity, both, or neither. List the discontinuities, if any exist.

1. $f(x) = \dfrac{x(x + 3)}{x + 3}$

2. $f(x) = \dfrac{2x}{x - 7}$

3. $f(x) = \dfrac{x - 5}{10}$

4. $f(x) = \dfrac{x - 4}{(x - 2)(x - 4)}$

5. $f(x) = \dfrac{x^2 - 3x}{x^2 - 9}$

6. $f(x) = \dfrac{x + 2}{x^2 - 6x - 16}$

C. Write a rational function $g(x)$ that matches the given characteristic(s).

1. Vertical asymptote at $x = 5$

2. Vertical asymptotes at $x = -2$ and $x = 1$

3. Vertical asymptotes at $x = 4$
Horizontal asymptote at $y = -3$

4. Vertical asymptotes at $x = -3$ and $x = 5$
Horizontal asymptote at $y = 1$

C. Write an example of a rational function that models each of the given characteristics.

1. A vertical asymptote at $x = -7$

2. A removable discontinuity at $x = 8$

3. A vertical asymptote at $x = 0$
A removable discontinuity at $x = -2$

4. A vertical asymptote at $x = -3$ and $x = 5$
A removable discontinuity at $x = 1$

IV. Add and Subtract Rational Expressions

A. Calculate the least common denominator (LCD) for each sum and difference. Describe any restriction(s) for the value of x.

1. $\dfrac{x}{3} + \dfrac{x+1}{15}$

2. $\dfrac{3x+4}{x} - \dfrac{5}{6x} + \dfrac{9}{2x}$

3. $\dfrac{x+1}{x^2 - 3x - 4} + \dfrac{x-3}{x-2}$

V. Multiply and Divide Rational Expressions

A. Multiply each expression. Describe any restriction(s) for the variables and simplify the answer when possible.

1. $\dfrac{2ab^2}{5c^3} \cdot \dfrac{15c}{4a}$

2. $\dfrac{x+3}{x-5} \cdot \dfrac{1}{x^2+6x+9} \cdot (x^2-25)$

3. $\dfrac{x^2-121}{x^2+x-20} \div \dfrac{x^2-10x-11}{x^2-25}$

VI. Problem Solving with Rational Functions and Equations

A. Solve each rational equation. Describe any restrictions for the value of x. Check your answer(s).

1. $\dfrac{2}{x} - \dfrac{3}{2x} = \dfrac{1}{x^2}$

2. $\dfrac{2}{x+3} + \dfrac{6}{x^2+3x} = \dfrac{1}{x}$

B. Write an equation to model each work scenario. Then, solve each equation.

1. Cleo can paint a room in 8 hours; while Phil can paint the same room in 6 hours. If they paint the room together, how long will it take them to paint the room?

2. The directions on the back of a 2-quart bottle of a 60% orange concentrate says it needs to be mixed with water to obtain a 20% orange drink. How much water should Hector add to the concentrate to obtain a drink that is 20% orange concentrate?

3. A research scientist has 10 liters of a 40% acid solution. She needs to create a solution containing 35% acid by mixing the 10 liters with a second solution containing 20% acid. How much of this second solution should she use?

4. Oni walked a half a mile to her sister's house to pick up her little brother and then walked back. The round trip took 60 minutes. If the rate at which she walked to her sister's house was 25% faster than the rate she walked while returning home, how fast did she walk on the way home?

5. An 8-man rowing crew rows at a speed of 10 miles per hour in still water. Every morning they practice by rowing 4 miles upstream and then 4 miles downstream. If it takes them $\frac{5}{6}$ of an hour to complete the trip, what is the speed of the current?