Asymptotes

Consider the graph of the function $f(x) = \dfrac{2x^2}{4 - x^2}$ in Figure 1.32.

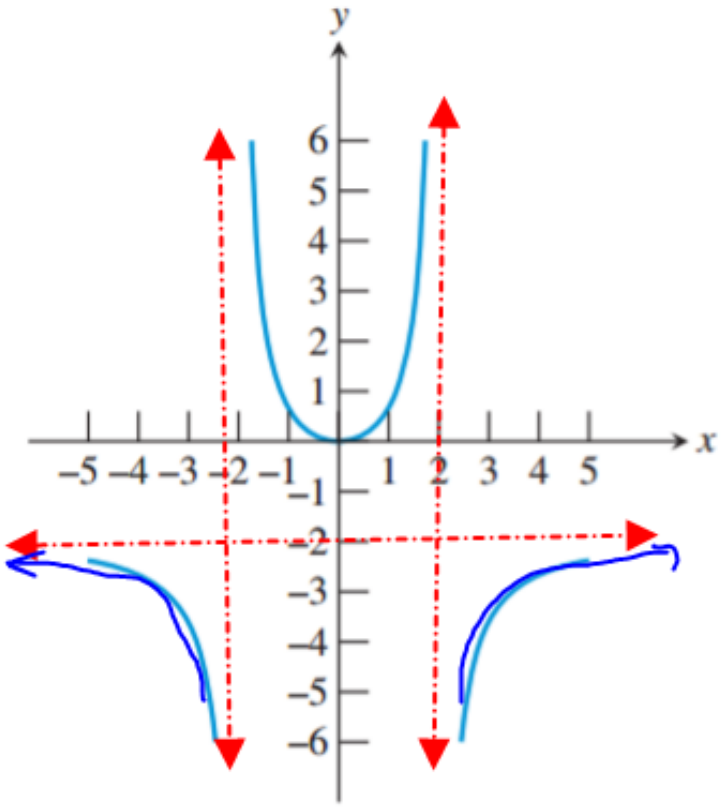


FIGURE 1.32 The graph of $f(x) = 2x^2/(4 - x^2)$ has two vertical asymptotes and one horizontal asymptote.

VA: $4 - x^2 = 0$

$\sqrt{x^2} = \pm\sqrt{4}$

$x = \pm 2$

HA: $\dfrac{2x^2}{-x^2} = -2$

$y = -2$

$\lim_{x \to +\infty} \dfrac{2x^2}{4 - x^2} = -2$

$\lim_{x \to -\infty} \dfrac{2x^2}{4 - x^2} = -2$

DEFINITION Horizontal and Vertical Asymptotes

The line $y = b$ is a **horizontal asymptote** of the graph of a function $y = f(x)$ if $f(x)$ approaches a limit of b as x approaches $+\infty$ or $-\infty$.

In limit notation:

$$\lim_{x \to -\infty} f(x) = b \quad \text{or} \quad \lim_{x \to +\infty} f(x) = b.$$

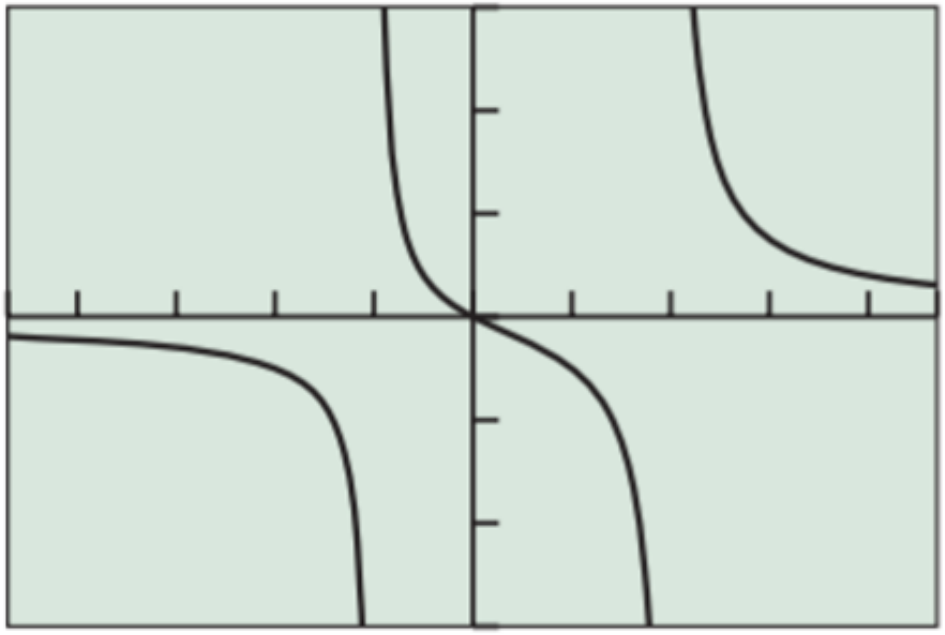
The line $x = a$ is a **vertical asymptote** of the graph of a function $y = f(x)$ if $f(x)$ approaches a limit of $+\infty$ or $-\infty$ as x approaches a from either direction.

In limit notation:

$$\lim_{x \to a^-} f(x) = \pm\infty \quad \text{or} \quad \lim_{x \to a^+} f(x) = \pm\infty.$$

Identifying the Asymptotes of a Graph

Identify any horizontal or vertical asymptotes of the graph of $y = \dfrac{x}{x^2 - x - 2}$.



[–4.7, 4.7] by [–3, 3]

DEFINITION Horizontal and Vertical Asymptotes

The line $y = b$ is a **horizontal asymptote** of the graph of a function $y = f(x)$ if $f(x)$ approaches a limit of b as x approaches $+\infty$ or $-\infty$.

In limit notation:

left right

$$\lim_{x \to -\infty} f(x) = b \quad \text{or} \quad \lim_{x \to +\infty} f(x) = b.$$

The line $x = a$ is a **vertical asymptote** of the graph of a function $y = f(x)$ if $f(x)$ approaches a limit of $+\infty$ or $-\infty$ as x approaches a from either direction.

$\lim_{x \to 2^-} \frac{2x^2}{4-x^2} = \infty$ $\lim_{x \to 2^+} \frac{2x^2}{4-x^2} = -\infty$

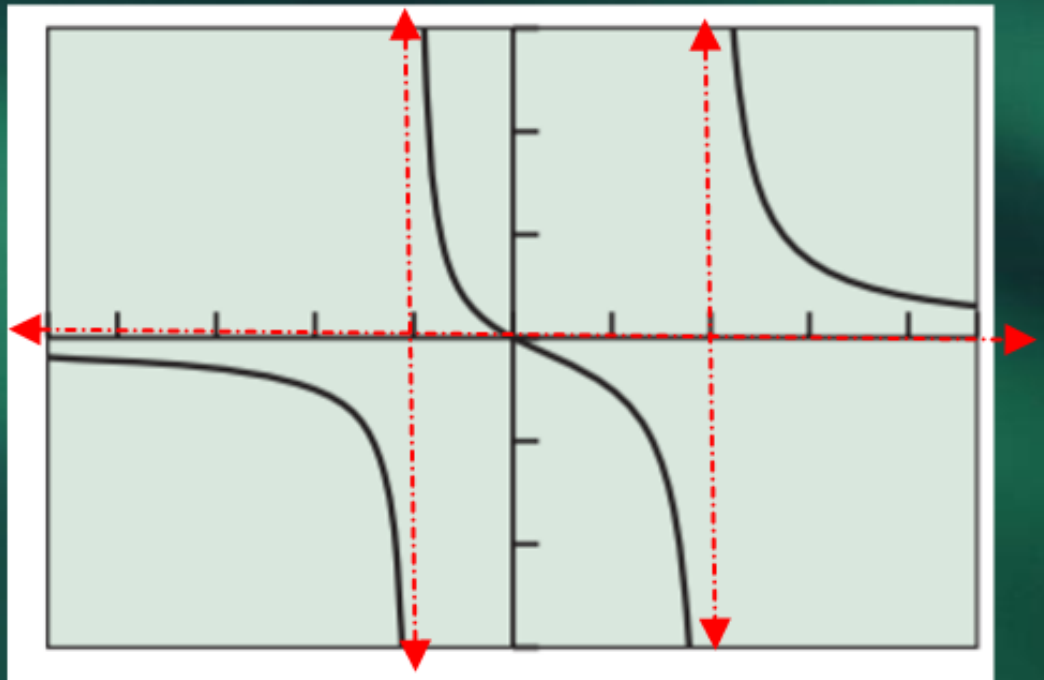
In limit notation:

$$\lim_{x \to a^-} f(x) = \pm\infty \quad \text{or} \quad \lim_{x \to a^+} f(x) = \pm\infty.$$

Identifying the Asymptotes of a Graph

Identify any horizontal or vertical asymptotes of the graph of

$$y = \frac{x}{x^2 - x - 2}.$$



[–4.7, 4.7] by [–3, 3]

VA: $x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2$ & $x = -$

HA $\frac{x}{x^2} = \frac{1}{x}$ y

$\lim_{x \to \infty} \frac{1}{x} = 0$

$x^2+1 \neq 0$

Matching Functions Using End Behavior

(a) $y = \dfrac{3x}{x^2+1}$ **(b)** $y = \dfrac{3x^2}{x^2+1}$ **(c)** $y = \dfrac{3x^3}{x^2+1}$ **(d)** $y = \dfrac{3x^4}{x^2+1}$

VA: NONE

HA: $\dfrac{3x}{x^2} = \dfrac{3}{x}$

$\lim_{x \to \infty} \dfrac{3}{x} = 0$

$y = 0$

VA: NONE

HA: $\dfrac{3x^2}{x^2}$

$y = 3$

VA: NONE

~~HA~~ $\dfrac{3x^3}{x^2}$

$3x$

$y = 3x$

$\lim_{x \to -\infty} 3x = -\infty$

VA: NON

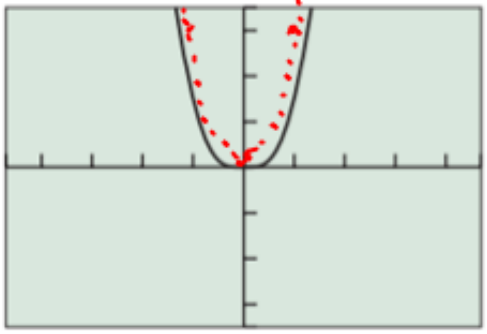
~~HA~~ 3

slant asymptote

$3x$

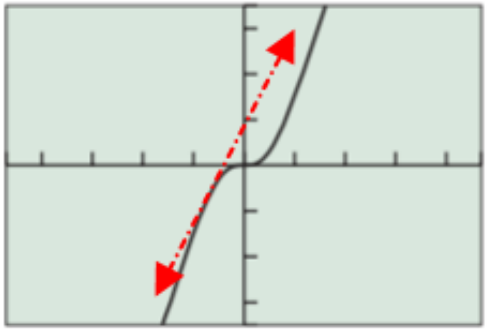
$\lim_{x \to \infty} 3x$

$\lim_{x \to -\infty}$



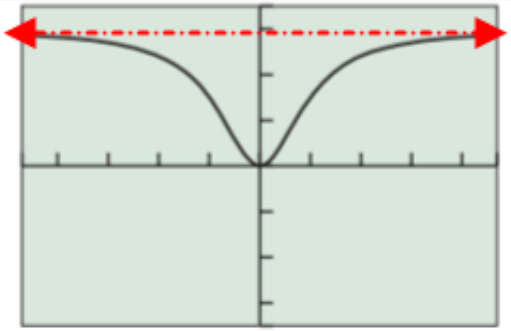
[–4.7, 4.7] by [–3.5, 3.5]

(i)



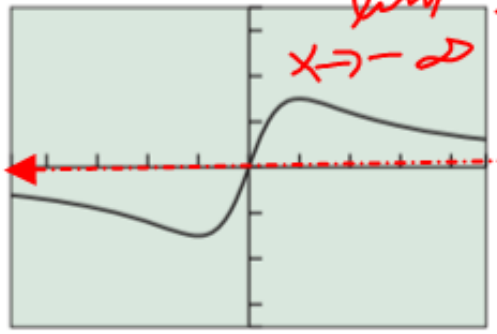
[–4.7, 4.7] by [–3.5, 3.5]

(ii)



[–4.7, 4.7] by [–3.5, 3.5]

(iii)



[–4.7, 4.7] by [–3.5, 3.5]

(iv)