Trigonometry Name:____________________Per:__

Fill in the table and leave in decimal form!

sin **Graphs**

x	$f(x) = \sin x$
0	
$\frac{\pi}{6}$	
$\frac{\pi}{4}$	
$\frac{\pi}{3}$	
$\frac{\pi}{2}$	
$\frac{3}{4}\pi$	
π	
$\frac{3\pi}{2}$	
2π	

1) Where does $\sin x = 0$?

These are points where $\sin x = 0$ crosses the $x-axis$. Graph them

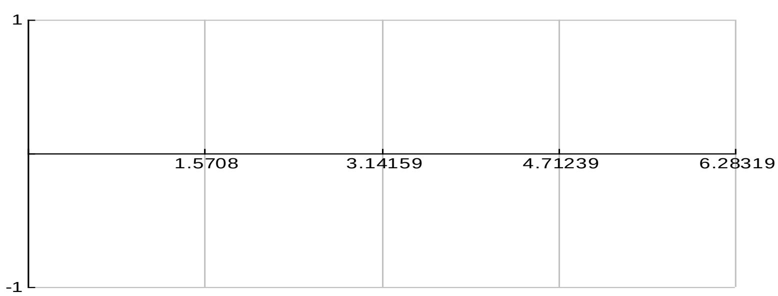
2) What is the maximum value of $\sin x$?
Graph the points where the max is.

3) What is the minimum value of $\sin x$?
Graph the points where the min is.

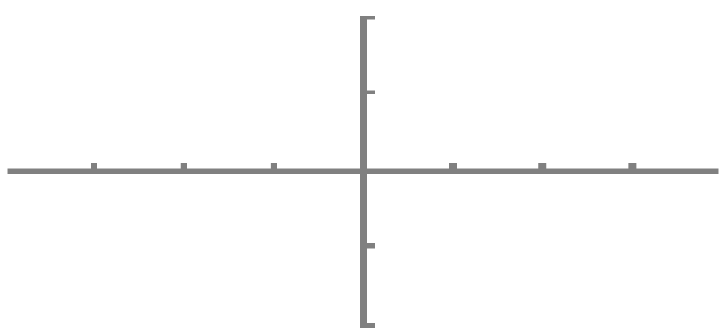
4) What is the range [$y-values$] of $\sin x$?

5) What is the domain [$x-values$] of $\sin x$?

6) Complete your graph of $\sin x$ below.



7) Graph and label the graph at right for $f(x) = \sin x$; on the interval $-360° \le x \le 360°$



Fill in the table and leave in decimal form!

$\cos$ **Graphs**

x	$f(x) = \cos x$
0	
$\frac{\pi}{6}$	
$\frac{\pi}{4}$	
$\frac{\pi}{3}$	
$\frac{\pi}{2}$	
$\frac{3}{4}\pi$	
π	
$\frac{3\pi}{2}$	
2π	

2) Where does $\cos x = 0$?

These are points where $\cos x = 0$ crosses the $x-axis$. Graph them

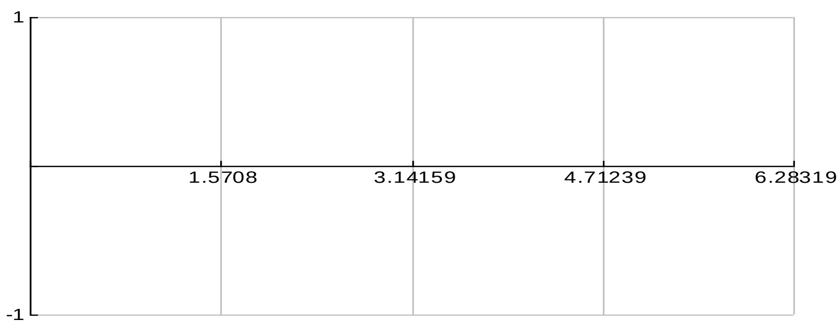
4) What is the maximum value of $\cos x$?
Graph the points where the max is.

5) What is the minimum value of $\cos x$?
Graph the points where the min is.

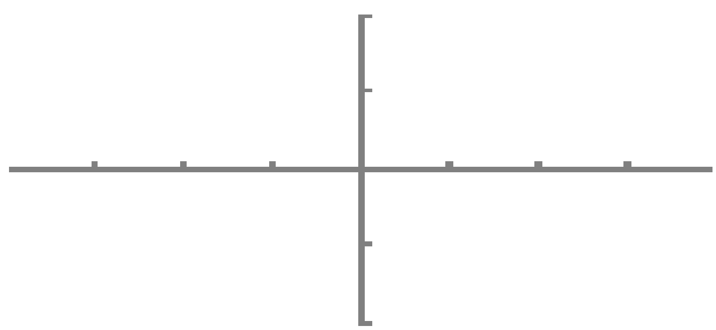
4) What is the range [$y-values$] of $\cos x$?

5) What is the domain [$x-values$] of $\cos x$?

6) Complete your graph of $\cos x$ below.



7) Graph and label the graph at right for $f(x) = \cos x$; on the interval $-360° \le x \le 360°$



$\tan$ & $\cot$ **Graphs**

1) Plot the points where $f(x) = \tan x$ crosses the $x - axis$
(where $\sin x = 0$)
2) Is $\tan x$ undefined anywhere?

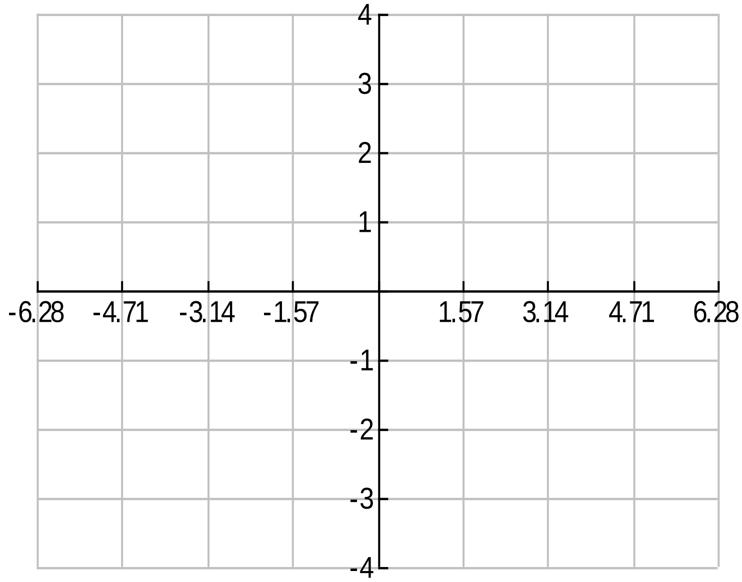
Draw asymptotes at these $x - values$
(where $\cos x = 0$)
3) Where is $\tan x$ positive?

4) Where is $\tan x$ negative?

5) What is the range of $\tan x$?

6) What is the domain of $\tan x$?

7) Plot the points for $\tan\left(\frac{\pi}{4}\right)$ and complete the graph for $f(x) = \tan x$



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2) Plot the points where $g(x) = \cot x$ crosses the $x - axis$
(where $\cos x = 0$)
7) Is $\cot x$ undefined anywhere?

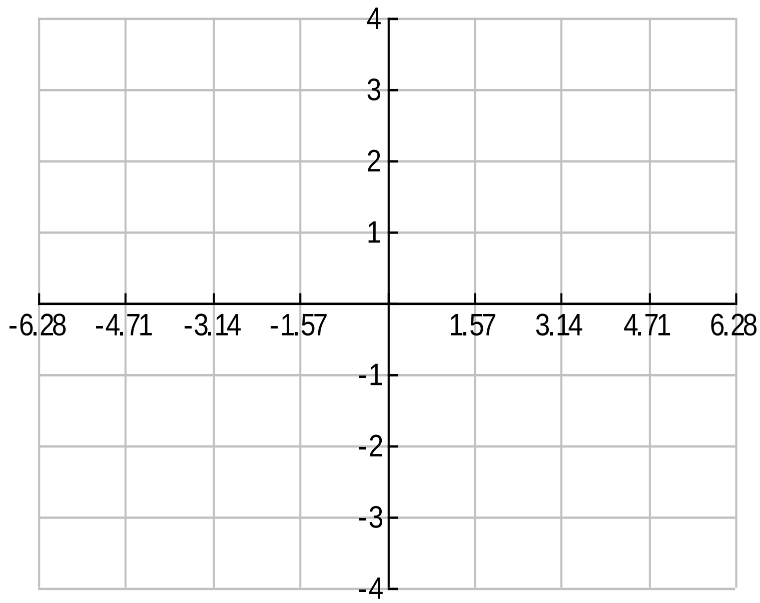
Draw asymptotes at these $x - values$
(where $\sin x = 0$)
8) Where is $\cot x$ positive?

9) Where is $\cot x$ negative?

10) What is the range of $\cot x$?

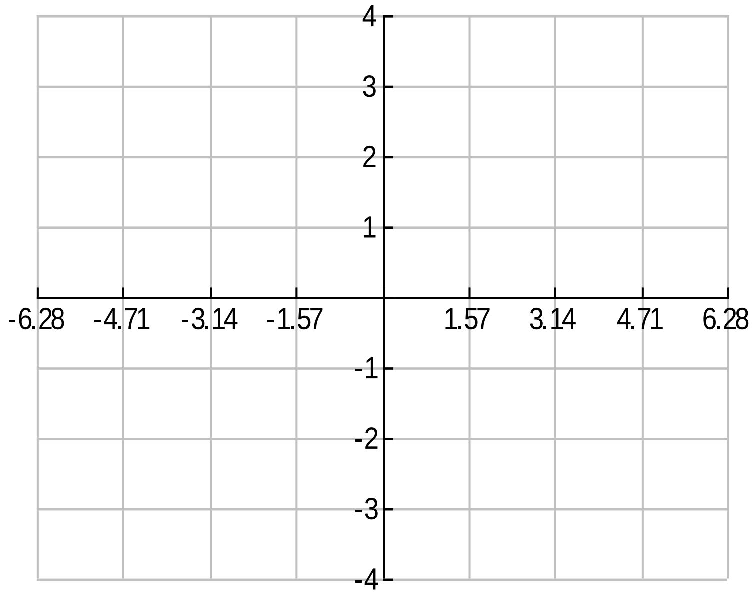
11) What is the domain of $\cot x$?

7) Plot the points for $\cot\left(\frac{\pi}{4}\right)$ and complete the graph for $g(x) = \cot x$



$\sec$ & $\csc$ **Graphs**

1) Graph $f(x) = \sec x$ in radians



2) Where is $\sec x$ undefined?

(where is $\cos x = 0$?)

3) Can $-1 < \sec x < 1$?

4) How often is $\sec x = 1$ on the interval $[-2\pi,\ 2\pi]$?

5) Does $\sec x$ have a minimum?

6) Does $\sec x$ have a maximum?

7) Draw in the vertical asymptotes with dotted lines

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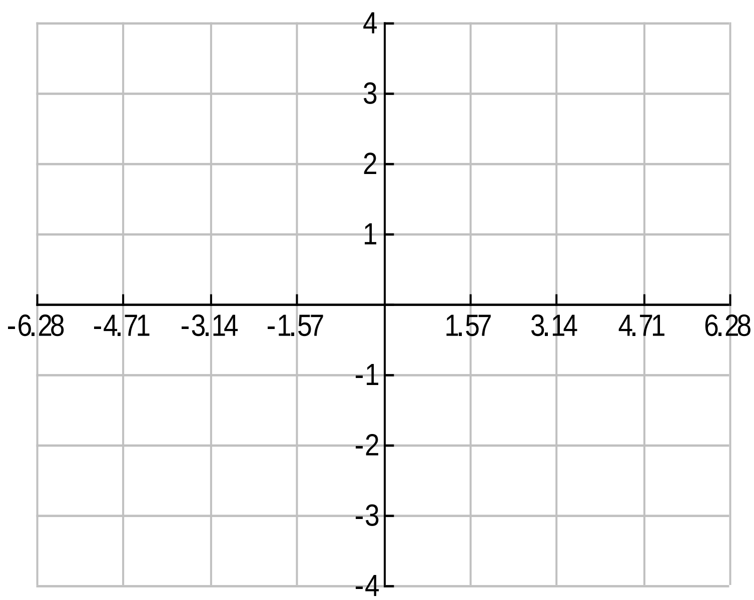
8) Graph $f(x) = \csc x$ in radians



9) Where is $\csc x$ undefined?

(Where is $\sin x = 0$?)

10) Can $-1 < \csc x < 1$?

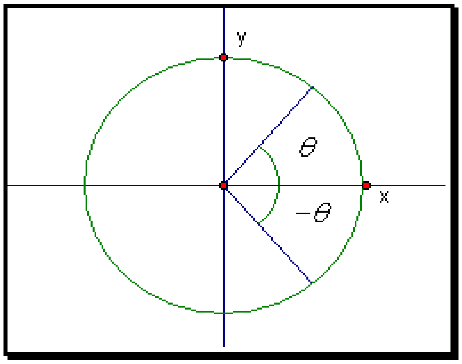
11) How often is $\csc x = 1$ on the interval $[-2\pi,\ 2\pi]$?

12) Does $\csc x$ have a minimum?

13) Does $\csc x$ have a maximum?

7) Draw in the vertical asymptotes with dotted lines

Trig Identities



$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\sec\theta = \frac{1}{\cos\theta} \qquad \csc\theta = \frac{1}{\sin\theta}$$

Note: In the unit circle diagram the difference between θ and $-\theta$. Both are on the right hand side (or positive) side of the $x-axis$. θ is on the positive side of the $y-axis$ and $-\theta$ is below, on the negative side.

1. Calculate each of the following

a) $\cos\left(\frac{\pi}{3}\right) =$ ________ b) $\cos\left(-\frac{\pi}{3}\right) =$ ________

c) $\cos\left(\frac{\pi}{4}\right) =$ ________ d) $\cos\left(-\frac{\pi}{4}\right) =$ ________

e) $\cos\left(\frac{5\pi}{6}\right) =$ ________ f) $\cos\left(-\frac{5\pi}{6}\right) =$ ________

g) How are each pair (a & b, c & d, e & f of the answers related to each other?

____

h) From the above answers you can conclude that $\cos(-\theta) =$ __________

2. Calculate each of the following

a) $\sin\left(\frac{\pi}{3}\right) =$ ________ b) $\sin\left(-\frac{\pi}{3}\right) =$ ________

c) $\sin\left(\frac{\pi}{4}\right) =$ ________ d) $\sin\left(-\frac{\pi}{4}\right) =$ ________

e) $\sin\left(\frac{5\pi}{6}\right) =$ ________ f) $\sin\left(-\frac{5\pi}{6}\right) =$ ________

g) How are each pair (a & b, c & d, e & f) of the answers related to each other?

____

h) From the above answers you can conclude that $\sin(-\theta) =$ __________

3. From our observations that $\cos(-\theta) = \cos\theta$ and that $\sin(-\theta) = -\sin\theta$ we can derive other identities. For example:

$$\tan(-\theta) = \frac{\sin(-\theta)}{\cos(-\theta)} = \frac{-\sin\theta}{\cos\theta} = -\tan\theta$$

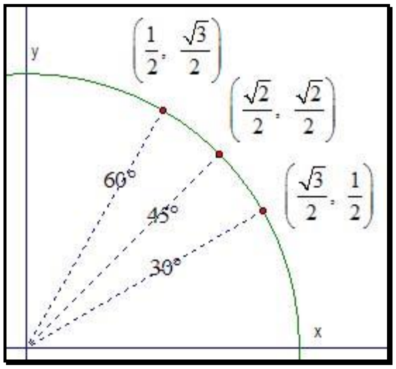
Complete the identities below:

a) $\sec(-\theta) = \frac{\quad}{\quad} =$

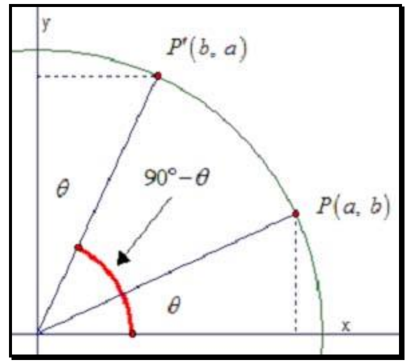
b) $\csc(-\theta) = \frac{\quad}{\quad} =$

c) $\cot(-\theta) = \frac{\quad}{\quad} =$

Cofunction Relationships



Note in the unit circle diagram above that the pattern repeats itself



Note the two congruent triangles whose vertices P and P' are shown. Witness how the x and y coordinates are switched

4a) Complete the following table

θ	$90° - \theta$	$\sin\theta$	$\cos(90° - \theta)$	$\sec\theta$	$\csc(90° - \theta)$	$\tan\theta$	$\cot(90° - \theta)$
0°							
30°							
45°							
60°							
90°							

4b) Fill out the identities below from your conclusions using the table above.

$\cos(90° - \theta) =$ ____________ $\csc(90° - \theta) =$ ____________ $\cot(90° - \theta) =$ ____________

$\sin(90° - \theta) =$ ____________ $\sec(90° - \theta) =$ ____________ $\tan(90° - \theta) =$ ____________

5. Pythagorean identities

$\boxed{\cos^2\theta + \sin^2\theta = 1}$

Recall that applying the Pythagorean Theorem on any angle θ in the unit circle gives us

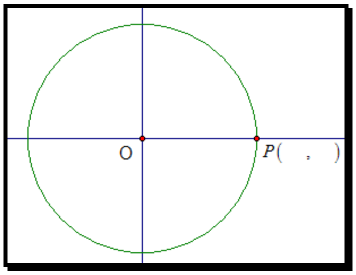
a) Hence: $\sin^2\theta = 1 - \cos^2\theta$ or $\cos^2\theta =$ ______________

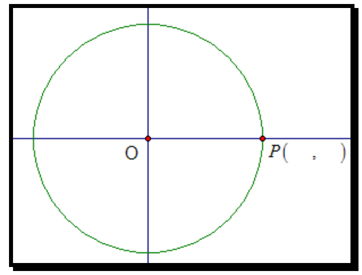
b) Divide every term on both sides of the boxed equation by $\cos^2\theta$ $\boxed{\sec^2\theta = \text{\_\_\_\_\_\_\_\_\_\_\_\_}}$

c) Divide every term on both sides of the boxed equation by $\sin^2\theta$ $\boxed{\csc^2\theta = \text{\_\_\_\_\_\_\_\_\_\_\_\_}}$

$$\cos(\alpha-\beta)$$

1) Draw angle α with terminal side in Quadrant II labeling the intersection with the **1st** unit circle as point A. Draw angle β with terminal side in Quadrant I, labeling the intersection with **1st** unit circle as point B.





2) Draw angle $\alpha-\beta$ with terminal side in Quadrant I labeling the intersection with the **2nd** unit circle as point C.

$\angle AOB \cong \angle ????$

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3) What are the coordinates in terms of trig functions for point $A(x, y)$?

(__________ , __________)

What are the coordinates in terms of trig functions for point $B(x, y)$?

(__________ , __________)

What are the coordinates in terms $(\alpha-\beta)$ of trig functions for point $C(x, y)$?

(__________ , __________)

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4) Substitute the coordinates for A and B into the distance formula to find the distance (A, B):

$$\sqrt{(\qquad - \qquad)^2 + (\qquad - \qquad)^2}$$

5. Foil the above, two square binomial expressions into two trinomials.

$$\sqrt{\qquad - \qquad + \qquad + \qquad - \qquad + \qquad}$$

6. Look for the identity $\sin^2\theta+\cos^2\theta=1$. Make substitutions for both occurrences.

$$\sqrt{\qquad - \qquad - \qquad}$$

7. Substitute the coordinates for C & P into the distance formula to find the distance (C, P)

$$\sqrt{(\qquad - \qquad)^2 + (\qquad - \qquad)^2}$$

8. Foil this like you did in step #5

$$\sqrt{\qquad - \qquad + \qquad + \qquad}$$

9. Look for and substitute the trigonometric Pythagorean Identity, then copy equation from #6 below

$$\sqrt{\qquad} = \sqrt{\qquad}$$

10. Square both sides on above equation; subtract 2 from both sides; then divide both sides by -2

$\cos(\alpha-\beta) =$ ______________________________

$$\cos(\alpha+\beta),\ \sin(\alpha+\beta),\ \sin(\alpha-\beta)$$

11. To find $\cos(\alpha+\beta)$, rewrite the expression as $\cos(\alpha-(-\beta))$

$\cos(\alpha-(\_\_\_\_))=\cos(\_\_\_\_\_)\cos(\_\_\_\_\_)+\sin(\_\_\_\_\_)\sin(\_\_\_\_\_)$

12. Substitute $\cos(-\theta)=\cos\theta$ identity and $\sin(-\theta)=-\sin\theta$ identity and rewrite the above equation

$\cos(\alpha+\beta)=$ ________________________

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13. To find $\sin(\alpha+\beta)$, substitute $\cos\left(\frac{\pi}{2}-\theta\right)=\sin\theta$ and $\sin\left(\frac{\pi}{2}-\theta\right)=\cos\theta$ with $\theta=(\alpha+\beta)$

$\sin(\alpha+\beta)=\sin(\theta)=\cos(\_\_\_\_-\_\_\_\_)=\cos\left(\frac{\pi}{2}-(\_\_\_\_+\_\_\_\_)\right)$

14. Distribute the negative sign and apply the associative property

$\cos((\_\_\_\_-\_\_\_\_)-\_\_\_\_)$

15. Apply cosine angle subtraction formula

$\_\_\_\_(\_\_\_\_-\_\_\_\_)\_\_\_\_(\_\_\_\_\_)+\_\_\_\_(\_\_\_\_\_-\_\_\_\_)\_\_\_\_(\_\_\_\_\_)$

16. Substitute $\sin\left(\frac{\pi}{2}-\theta\right)=\cos\theta$ and $\cos\left(\frac{\pi}{2}-\theta\right)=\sin\theta$ identities and rewrite above equation

$\sin(\alpha+\beta)=$ ____________________________

---

17. To find $\sin(\alpha-\beta)$, substitute $-\beta$ for β in $\sin(\alpha+\beta)$ formula.

$\sin(\alpha+(-\beta))=$ _______________________________

18. Substitute using identities from the method in #12

$\sin(\alpha-\beta)=$ ____________________________

Summarize: Sine and Cosine Angle Addition/ Subtraction formulas

$\sin(\alpha\pm\beta)=$

$\cos(\alpha\pm\beta)=$

$\tan(\alpha+\beta)$

Now to derive the formula for $\tan(\alpha+\beta)$.

1. First, fill in the formulas for: $\sin(\alpha+\beta) =$ ______________________________

   $\cos(\alpha+\beta) =$ ______________________________

2. In terms of sin and cos, $\boxed{\tan\theta = \underline{\hspace{2cm}}}$ therefore $\tan(\alpha+\beta) = \underline{\hspace{2cm}}$

3. Substitute your formulas from #1 in the numerator and denominator of the quotient from above expression

$$\tan(\alpha+\beta) = \frac{\qquad + \qquad}{\qquad - \qquad}$$

4. Divide every term in the top and bottom by $\cos\alpha\cos\beta$

$$= \frac{\underline{\qquad} + \underline{\qquad}}{\underline{\qquad} - \underline{\qquad}}$$

5. Look for terms that reduce to "1" or cancel

$$= \frac{\underline{\qquad} + \underline{\qquad}}{- \quad \underline{\qquad}\ \underline{\qquad}}$$

6. Rewrite each $\frac{\sin}{\cos}$ [of the same angle only] as tan of that angle. This is the desired formula.

$$\tan(\alpha+\beta) = \boxed{\qquad\qquad\qquad}$$

7. Find the exact value for $\tan\left(\frac{5\pi}{12}\right)$ in simplest radical form. Then verify your answer with your calculator

$$\tan(\alpha-\beta)$$

Now get the formula for $\tan(\alpha-\beta)$

8. First fill in the formulas for

$$\sin(-\theta) = \_\_\_\_\_\_$$
$$\cos(-\theta) = \_\_\_\_\_\_$$

9. Use the negative formulas to find $\tan(-\theta) = \frac{\quad}{\quad} = \frac{\quad}{\quad} = \_\_\_\_\_\_\_\_\_\_$

Copy down formula derived earlier for $\tan(\alpha+\beta) =$

And rewrite it as $\tan(\alpha+(-\beta))$

$$\tan(\alpha+(-\beta)) = \frac{\qquad\qquad}{\qquad\qquad}$$

Substitute $-\tan\beta = \tan(-\beta)$ where appropriate

$$\tan(\alpha+\beta) = \boxed{\qquad\qquad\qquad\qquad}$$

Find the tangents in simple radical form and verify the answer with your calculator

10. $\tan\left(\frac{\pi}{12}\right)$

11. $\tan(\theta+\pi)$

Double Angle Formulas

1. Fill out each of the formulas we have derived so far:

$\cos(\alpha \pm \beta) =$

$\sin(\alpha \pm \beta) =$

$\tan(\alpha \pm \beta) =$

2. To find the formulas for $\sin(2\alpha)$, use the $\sin$ addition formula and set $\beta = \alpha$

$\sin(\alpha + \alpha) =$ ____________________________

$\sin(2\alpha) =$

3. To find formulas for $\cos(2\alpha)$ use the $\cos$ addition formula and set $\beta = \alpha$

$\cos(\alpha + \alpha) =$ ____________________________

$\cos(2\alpha) =$

4. Greg dropped his calculator and now his "cos" button won't work. How can he find $\cos(2\alpha)$?

Hint: $\sin^2\theta + \cos^2\theta = 1$

$\cos(2\alpha) =$

5. Marsha hit Greg over the head with her calculator and now her "sin" button won't work. How can she find $\cos(2\alpha)$?

$\cos(2\alpha) =$

6. Peter was taking a test but his y^x key and his multiplication key on his calculator broke and he had to find $\cos^4(15°) - \sin^4(15°)$. What will he punch into his calculator to get the right answer?

Hint: $a^2 - b^2 = (a+b)(a-b)$

7. Not to be outdone by Marsha, Jan used the method of #2 and #3 to derive a formula for $\tan(2\alpha)$ in terms of the tangent function. Help Jan find her $\tan$.

$\tan(\alpha + \alpha) =$ ____________________________

$\tan(2\alpha) =$

Half Angle Formulas

1a) Fill out all three of the double angle formulas for:

$\cos(2\theta) =$ ______________________ $=$ ______________________ $=$ ______________________

1b) Let $x = 2\theta$. Solve this for θ. Then $\boxed{\theta = \text{\_\_\_\_}}$

2. Substitute the identities in step 1b) into the double angle formula that only has "cos" in it, so that x is the only variable.

$\cos(x) =$

Add 1 to both sides ⟶

$=$

Dived both sides by 2 ⟶

$=$

Take the square root of both sides ⟶

$=$

3. Substitute the identities in step 1b) into the double angle formula that only has "sin" in it, so that x is the only variable.

$\cos(x) =$

Subtract 1 from both sides ⟶

$=$

Dived both sides by -2 ⟶

$=$

Take the square root of both sides ⟶

$=$

4.

$\boxed{\pm\sqrt{\dfrac{1+\cos x}{2}} =}$ $\boxed{\pm\sqrt{\dfrac{1-\cos x}{2}} =}$

$$\tan\left(\frac{x}{2}\right)$$

1a) Fill out the double angle formulas for:

$\boxed{\tan(2\theta) = \qquad}$

1b) And the half-angle formulas for:

$\boxed{\sin\left(\frac{\theta}{2}\right) = \qquad}$

$\boxed{\cos\left(\frac{\theta}{2}\right) = \qquad}$

1c) Let $x = 2\theta$. Then $\theta = \frac{\quad}{\quad}$

2a) Write $\tan$ as $\sin$ over $\cos$

$$\tan\left(\frac{\theta}{2}\right) = \frac{\qquad}{\qquad}$$

2b) Substitute the half angle formulas for $\sin$ over $\cos$

$$= \pm\frac{\sqrt{\frac{\qquad}{\qquad}}}{\sqrt{\frac{\qquad}{\qquad}}}$$

2c) Put under one giant square root sign, then "*cancel*" any common factors

$$= \pm\sqrt{\frac{\qquad}{\qquad} \cdot \frac{\qquad}{\qquad}}$$

Formula #1

$\boxed{= \tan\left(\frac{x}{2}\right) = \qquad}$

3a)Multiply top and bottom of $\tan\left(\frac{x}{2}\right)$ by $\sqrt{1-\cos x}$

$$\sqrt{\frac{\qquad}{\qquad}} \quad \sqrt{\frac{\qquad}{\qquad}}$$

3b)Don't foil the product. Use difference of two squares

$$\sqrt{\frac{\qquad}{\qquad}}$$

3c)Substitute Pythagorean Identity for $1-\cos^2 x$

$$\sqrt{\frac{(\qquad - \qquad)^2}{(\qquad)^2}}$$

Formula #2

$\boxed{\tan\left(\frac{x}{2}\right) = \qquad}$

4a)Multiply top and bottom of $\tan\left(\frac{x}{2}\right)$ by $\sqrt{1+\cos x}$

$$\sqrt{\frac{\qquad}{\qquad}} \quad \sqrt{\frac{\qquad}{\qquad}}$$

4b)Don't foil the product. Use difference of two squares

$$\sqrt{\frac{\qquad}{\qquad}}$$

4c)Substitute Pythagorean Identity for $1-\cos^2 x$

$$\sqrt{\frac{(\qquad)^2}{(\qquad + \qquad)^2}}$$

Formula #3

$\boxed{\tan\left(\frac{x}{2}\right) = \qquad}$

Formulas for $\cot(\alpha+\beta)$, $\cot(2\theta)$, $\cot\left(\frac{x}{2}\right)$

1a) Fill in the missing parts

$$\cot(\alpha+\beta) = \frac{1}{\rule{4cm}{0.4pt}} = \frac{\rule{4cm}{0.4pt}}{\dfrac{\quad + \quad}{\quad - \quad}} = \frac{\quad - \quad}{\quad + \quad}$$

1b)Rewrite each tangent in terms of cot $\left(\tan = \frac{1}{\cot}\right)$

1c)Find common denominator

1d)Cancel common terms

$$\frac{1 - \dfrac{1}{\rule{2cm}{0.4pt}}}{\dfrac{1}{\rule{1.5cm}{0.4pt}} + \dfrac{1}{\rule{1.5cm}{0.4pt}}} = \frac{\dfrac{\quad - \quad}{\rule{2cm}{0.4pt}}}{\dfrac{\quad + \quad}{\rule{2cm}{0.4pt}}} = \rule{4cm}{0.4pt}$$

2 Substitute $-\beta$ for β and $\cot(-\theta)$ for $-\cot(\theta)$ into cot addition formula to derive difference formula

$\cot(\alpha-\beta) =$

3a) Derive a double angle formula for the tangent by using the angle addition formula in # 1

$\cot(2\theta) =$

3b) Since $\cot = \frac{1}{\tan}$, write the three different half angle formulas for the cotangent

$$\cot\left(\frac{x}{2}\right) = \rule{4cm}{0.4pt} = \rule{4cm}{0.4pt} = \rule{4cm}{0.4pt}$$

4a) Using the $\cos\left(\frac{x}{2}\right)$ formula, derive a formula for $\sec\left(\frac{x}{2}\right)$

4b) Modify the formula so there is no $\sqrt{\quad}$ in the denominator [hint: conjugate]

5a) Using the $\sin\left(\frac{x}{2}\right)$ formula, derive a formula for $\csc\left(\frac{x}{2}\right)$

5b) Modify the formula so there is no $\sqrt{\quad}$ in the denominator [hint: conjugate]